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Journal of the

STRUCTURAL DIVISION

Proceedings of the American Society of Civil Engineers

EFFECTS OF RESEARCH ON MODERN AMERICAN STRUCTURAL CONCRETE DESIGN

Eivind Hognestad,[1] M. ASCE, M. IABSE

This paper is one of the group of papers which were the bases for oral presentations at the Joint ASCE-IABSE Meeting, New York, October 1958. The entire group, including those published in CIVIL ENGINEERING, will be reprinted in one volume.

SYNOPSIS

The organization and the nature of recent structural concrete research in the United States are outlined. Research approaches used in the past resulted in substantial design improvements. The need for a shift from applied to basic research in the future is stressed. A bolder and more imaginative course in future research seems necessary to provoke the radically new ideas needed for future improvements in concrete design.

Scope

Structural concrete design is a combination of science and art molded together by the designer's insight and experience as a builder. So it has been, so it must remain. Modern structural design would be unthinkable without a base of science, but it would also stagnate without the dreams and beauty of art. The human characteristics of the builder—his enthusiasm, imagination, and experience—are needed to mold a balanced whole. For concrete design is both architecture and engineering; as such it is dedicated to the welfare of man in a material sense as well as in a spiritual sense.

Being based on science, American structural concrete design has, of course, been profoundly influenced by scientific advances through research

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from the pioneering work at the turn of the Century and up to the present day. The technical nature of this influence has varied in detail and shall not be summarized here. With the aid of typical examples the organization and the nature of recent structural concrete research in the United States will be outlined. It is found that the research approaches used in the past resulted in substantial design improvements. But an even bolder and more imaginative course in future research seems necessary to provoke the radically new and yet unborn ideas destined to color our structural concrete design of the future.

Organization of Research

By its very nature, the research of a nation cannot be rigidly controlled or planned by a few minds. But guidance, coordination, and a forum for interchange of thought and ideas are prerequisites for orderly research progress. The key is cooperation between many men and organizations.

Guidance

Ever since its formation over 50 years ago, the American Concrete Institute has been the principal guiding force in American concrete research, including the structural field. A large portion of our research literature is found in the fifty-five volumes of the ACI Journal. Technical committees initiating research have been in existence for many years, and the ACI has performed a vital function through committees translating research into such documents as "Building Code Requirements for Reinforced Concrete" and "Manual of Standard Practice for Detailing Reinforced Concrete Structures". Finally, the Annual Conventions and Regional Meetings of the Institute have offered opportunities for individuals of varied backgrounds to meet in a forum of cooperation.

The American Society of Civil Engineers has for many years contributed to the development of structural concrete. During the past decade, the guidance of structural concrete research provided by the ASCE has been substantially improved and expanded. A series of technical committees dealing with research and the translation thereof into design practice have been established jointly by the Structural Division of ASCE and the American Concrete Institute—again an example of fruitful cooperation. During the past year, the Committee on Research responsible to the ASCE Board of Directors has become dynamically active toward its mission of reviewing and coordinating research activities of the Society. Finally, ASCE conventions and conferences contribute immeasurably toward research and effects thereof in engineering practice.

Execution

Educational institutions have been the backbone in execution of American structural concrete research ever since its inception at the turn of the century. Thirty-nine such institutions reported active work in progress in 1958. In addition, twenty-nine trade associations and public agencies had concrete research in progress. Some of these non-educational institutions, such as the Association of American Railroads and the Portland Cement Association, have recently expanded their structural research facilities and activities greatly,

as the need for new knowledge and the type of experimentation required have grown beyond what our universities can efficiently do alone.

Sponsorship

A major portion of American structural concrete research at educational institutions is being financed by various agencies of federal and state governments, notably the Department of Defense, the National Science Foundation, and the Bureau of Public Roads in cooperation with various State Highway Departments. Various groups of the construction industry are also financing structural concrete research, and funds or faculty services are frequently contributed by educational institutions toward their own research work.

The Reinforced Concrete Research Council

It is difficult to select a single example indicating the cooperative nature of American structural concrete research. But the activities of the Reinforced Concrete Research Council are typical and have given results that command for the Council a position of leadership.

In the early 1940's, a need was recognized in this country for a new theory of structural concrete design, a need to discontinue progress based solely on periodic adjustments and modifications of the classical straight-line theory, a need for a fundamental change to a new theory based on the actual inelastic properties of concrete and reinforcing steel. A Subcommittee of the Committee on Masonry and Reinforced Concrete was formed in the Structural Division of the American Society of Civil Engineers in 1944. It immediately commenced a study of the adequacy of the new inelastic design theories, and it became evident that a great deal of experimental research was needed.

Such was the situation when the Reinforced Concrete Research Council was organized in 1948 under the inspiring chairmanship of the late Robert F. Blanks. Fundamental research in this country and elsewhere had brought into being the nucleus of a new design theory known as ultimate strength design. To develop this theory to a stage of practical usefulness these were needed: (1) an organized cooperative effort, (2) scientific planning and guidance, and (3) economic support of experimental research. In short, a hard push forward was required, a push that could not be accomplished by any single one of the interests involved.

The formation of a Research Council took place early in 1947, and the organization and financing of the Council were completed about a year later. The Council is an independent organization under the sponsorship of the Engineering Foundation. It consists of some two dozen representatives of technical societies, government agencies, trade associations, outstanding engineering organizations, and several universities. The Council obtains funds from the Engineering Foundation and several interested organizations, so that each contributor receives many times the research value of his individual contribution. The total value of research sponsored by the Council approached one-half million dollars in 1948-58. Furthermore, this research work is guided by the Council membership, and small task committees carefully follow each individual investigation or project. The actual project work has usually been done in universities, which through Council sponsorship not only get a research job done well, but also make a most valuable contribution to our Nation's future by training the students to become scientists and engineers

of tomorrow. The results of projects sponsored by the Council are published in technical journals and also in a series of Bulletins available free from the Engineering Foundation.

The immediate broad objective of the Council at the time of its formation was to re-examine critically the basis for reinforced concrete design methods, and to develop a new theory of design into a workable form intended to replace the elastic concepts that have been in use for many years. To reach this goal, many research projects were initiated, sponsored, and carried to a successful completion. Details of those projects will not be dwelt upon here, but a deeper question is raised: "What were the final effects on structural concrete design?"

In 1955, a Joint Committee on Ultimate Strength Design of the American Society of Civil Engineers and the American Concrete Institute submitted a final report completing its assignment "to evaluate and correlate theories and data bearing on ultimate strength design procedures with a view to establishing them as accepted practice". These Joint Committee studies were based largely on experimental data and theories obtained through Reinforced Concrete Research Council sponsorship. The Building Code Committee of the American Concrete Institute then took a historical step stated in one and one-half lines under Design Methods. This statement now appears in the 1956 ACI Building Code as "The ultimate strength method of design may be used for the design of reinforced concrete members". For ready reference, an abstract of the Joint Committee Report was appended to the Building Code. Research in the field of ultimate strength design has continued under Council sponsorship, and major expansion and improvement of the new methods may be expected when another revision of the ACI Building Code appears.

In this manner, the immediate broad objectives of the Reinforced Concrete Research Council were accomplished, and the Council turned to other tasks. Already soon after the initial projects were underway, it became evident that the Council offered an unusual opportunity to foster, correlate and sponsor other needed research in reinforced concrete. A Joint Committee on Shear and Diagonal Tension of the American Concrete Institute and the American Society of Civil Engineers was formed in 1950. A chain of research projects was planned, sponsored, and completed as a result of cooperation between this committee and the Council. In 1958 the Joint Committee on Shear and Diagonal Tension began preparation of its final report to complete its assignment "to develop methods for designing reinforced concrete members to resist shear and diagonal tension consistent with the new ultimate strength design methods." Again, an important report is being based largely on test data and theories developed through Council sponsorship.

Similar cooperation with technical committees has led to projects directed toward improved design methods for reinforced concrete floor and bridge slabs. The Council is also sponsoring research in prestressed concrete, folded plate structures, and several other fields. It has been found that the Council can also function to shorten the lag between research and practice by acting as liaison between research agencies and technical committees of the American Society of Civil Engineers, the American Concrete Institute, and similar organizations. The scope of activities of the Council has accordingly been broadened to encompass the entire field of reinforced concrete research.

Mileposts of the Past

Structural concrete research of the future must be aimed boldly toward major technological breakthroughs. Marginal progress is not enough in an age when serious consideration is being given to construction of airports on the moon, when man is on the verge of harnessing the power of the sun and atomic fusion. To plan the future it is helpful to examine the past, and several mileposts of progress are found:

(1) The invention of reinforced concrete appears to have been the product of a period rather than a few individual minds. However, the breakthrough in its development came when a design theory based on the science of engineering mechanics was formulated in France and Germany shortly before 1900.

(2) The discovery of the water-cement ratio principle was an outstanding example of a gradual scientific development, reaching the breakthrough stage shortly before 1920.

(3) The development of reinforced concrete shell structures and theories for their design began in Germany in the early 1920's. This important development resulted from the pioneering work of a few individuals.

(4) Prestressing was attempted already at the turn of the century. However, the milepost was not established before creep and shrinkage properties of concrete and the need for high-strength reinforcing steels were recognized about 1930.

(5) Remarkable improvements in the durability of concrete structures through air entrainment became a practical reality in the late 1930's. This development was carried into practice by an unprecedented volume of research in America and abroad.

(6) The modern type of deformed reinforcing bar was developed independently in Sweden and the United States about 1940, as a result of a series of research efforts dating back as far as 1910.

From these six examples of the past can be learned two major truths: First, milestones were usually established by cooperation extending throughout the world. Secondly, it often took discouragingly long to develop a new idea into practical reality. Tradition, conservatism, and plain human resistance to change were formidable obstacles on the road to progress. There is developing among structural engineers a yearing for new and better methods and materials unprecedented in the history of their profession. This feeling is reflected by the strong emphasis on research now developing in the American Society of Civil Engineers.

American Research in Progress

Triggered by research, American structural concrete design is in a process of rapid change. Present research activity is facing a wide field of fairly well defined problems urgently in need of solution to advance practice. Inventive genius and fundamental research of the past have resulted in a pronounced need for applied research to bridge the gap between basic ideas and improvements in the arts of practice.

Shell structures and prestressing are already established historical mileposts. But a large volume of research remains to be done—and is being done —to expand and improve their applications.

Ultimate strength design, that is design of cross sections on an inelastic basis, became a practical reality in America with the 1956 ACI Building Code. However, intense studies of shear, deflection, and flexural crack formation are in progress to broaden and improve this new design concept. A new Joint ASCE-ACI Committee on Limit Design was organized recently to guide research directed toward extending inelastic design concepts beyond the proportioning of sections and into the field of moment analysis of indeterminate frame systems.

Applications of ultimate strength design have already led to two trends that hold great promise for the future. By its realistic and accurate appraisal of the structural performance of unusual concretes and reinforcing steels, ultimate strength design has facilitated a trend toward the use of high-strength concretes and reinforcing steels. The American Society for Testing Materials is now in the process of revising reinforcing steel specifications. New steels with minimum yield points of 60,000 and 75,000 psi will probably be included in new specifications. This development, and the companion trend toward high-strength concretes, are pointing toward the light, graceful, and economical reinforced concrete buildings and bridges of tomorrow. Several research projects are in progress to speed this development.

A second trend is related to the rapid growth of a new industry, the precast concrete industry. The potentialities of precasting certainly go far beyond the mere concept of casting a building member at ground level rather than several stories up in the air, or in a factory on a river bank instead of in midstream. These potentialities result in large part from the advantages of mechanized mass-production methods that permit repeated use of forms and equipment, as well as the strict controls of manufacture necessary to use high quality materials to the best economic advantage. Careful design down to the smallest detail becomes of great importance when a member is to be made not once, but dozens and even hundreds of times; when heavy investments go into a mechanical manufacturing process. Therefore, there is hardly any other way of designing for mass production than to use every trick available in the field of structural concrete technology. Only the best and most refined design procedures are good enough to mass produce products that will perform perfectly the function intended in design and still be as economical as possible.

The development of the precasting industry has led to a new need for knowledge of details in structural concrete technology. For example, an entire new branch of technology involving connections between precast units has come into being. Research in this field is in progress at the Portland Cement Association Laboratories, but there is a most pressing need for the ideas and efforts of others. A field even more complex than connections in structural steel must be developed, and be developed fast.

The American lightweight aggregate producing industry is expanding rapidly. Most of the 41 plants producing lightweight aggregates from clays and shales have been built within the last decade. Eighteen plants are in operation producing lightweight aggregate from slag. Increasing demands for light weight, coupled with high strength, have established an important position for lightweight aggregates in American structural concrete. Again, an industrial development triggered by research has led to a new branch of structural concrete technology; and further research is in progress to develop a complete theory of lightweight-aggregate structural concrete design.

The last field of research in progress to be mentioned here is possibly a most important one. It has been customary for many years to provide for a wide variety of uncertainties in design and construction by a single safety or load factor. Thus, even in ultimate strength design, which is vastly superior in this respect to the straight-line theory, service loads times load factors are equated to a strength function involving properties of the member concerned and the materials used. However, the strength function of beams is frequently controlled by yielding of the reinforcing steel so that an accidental reduction in concrete strength hardly reduces the strength at all. On the other hand, the strength function of some columns is influenced by variations in concrete strength almost to the extent of proportionality. To remedy this unbalance, the safety concept may be split into two: load factors only slightly greater than one to provide for accidental overloads, and uniformity factors smaller than one to allow for variations in the strength of concrete and reinforcement below those specified. Such a change in design philosophy could make possible substantial reductions in sections and costs without danger to the safety of concrete structures. A minor increase in costs of improved control and inspection could return considerable savings.

These researches in progress in the fields of ultimate strength and limit design, high-strength deformed bar reinforcement, precasting, lightweight aggregate structural concretes, and safety concepts are examples of developments clearly destined to have profound effects on future American structural concrete design. Some of these developments will probably become recognized in the future as major mileposts of progress.

Research for the Future

The past and the present of structural concrete research in America has been reviewed. It seems fitting to close this discussion with a look toward the research of the future.

Research results are today being translated into practice at a rate unequaled in the history of structural concrete. The chains of tradition have been broken, and most gratifying progress is being made toward better and more economical concrete structures. But this high-crested wave of activity rose from several decades of basic research carried out slowly and on a small scale in our Nation's universities and abroad. The wave seen today must eventually die down. New waves of the future can evolve only from basic research, from investigations in which immediate practical needs are subordinated to long-range benefits.

It seems probable that American structural concrete research can run profitably along its present major path of highly applied studies only into the early 1960's. Beyond that time research must gradually turn to fundamental studies; not only to the common type of fundamental research that is directed inward to a deeper understanding of structural concrete materials as known today; but also to fundamental studies of shape and form conducted with boldness, imagination, and enthusiasm toward the radically new ideas yet undeveloped or even unborn. At least three of these mileposts of the future can now be seen in the haze of the research horizon.

French, Russian, and a few American experiments have indicated that it should be feasible to produce a new type of concrete that hardens to considerable strength as concretes of today do. Then, a controlled expansion by

special cements or special aggregates should take place in the solid state to prestress embedded high-strength reinforcement in all directions of space. Following this controlled expansion, the material should remain stable throughout its useful life of service. To produce such a radical step forward in concrete technology may take many years of fundamental study in chemistry and solid state physics, but the goal seems possible to reach.

As far as curved structures in space are concerned, only a modest beginning has been seen. Combinations of shells, box-sections, and three-dimensional frameworks may open great new possibilities in future concrete construction.

Cellular concretes light as cork with fine reinforcing fibers of steel or glass may in the future provide adequate strength, durability, and insulation in a single skin structure of tremendous span made up of small, factory-made units.

Beyond these three dreams on the horizon, there must be a vista of things to come beyond our imagination. Man cannot sit by waiting for the future to come. There is yet much suffering and want in this world. It is the duty of engineers and scientists to progress vigorously within their fields of service to the welfare of man.

The Road Ahead

Those who wish to fulfill their obligation to the future may feel that the type of research required cannot find sponsorship, cannot be financed. This is probably not so. During a conference on research organized by the American Society of Civil Engineers at George Washington University in September 1958 several discussion groups arrived independently at the conclusion that there exists a shortage of imaginative research proposals well prepared by competent men. The National Science Foundation reports that in 1953-54 American industry as a whole channeled 1.7 per cent of sales into research and development, some industries going as high as 6 to 9 per cent. If an ample flow of creative ideas existed, certainly the $50 billion American construction industry could profitably invest one per cent in the future. The resulting 500 million dollars per annum would sustain 10,000 good-sized research programs at $50,000 each. There is a shortage of creative ideas, not of potential sources of funds.

To improve the production of ideas and their development through sponsored research, there are many things that can be done, and it is most gratifying that the American Society of Civil Engineers is recognizing its duty to encourage, guide and aid research. To do this as a united profession of scientists and engineers in cooperation, communications must be improved.

The traditional form of scientific report, dissertation, or paper, is a communication addressed to fellow scientists. Such communication is often expressed in considerable detail and in highly guarded, cautious language. This type of communication has served and will always serve a definite purpose as a step of the process of development within science itself.

However, in recent years it has become increasingly evident in the United States that the great majority of our engineers, busily occupied in the important and daily task of building a better America, cannot find the time to devote thorough study to the rapidly increasing volume of scientific written communications. Some American scientists, therefore, share a view that

improved communications must be developed to place new developments in the hands of engineers and architects in a position to put them to practical use, and to create a better feed-back of new ideas from practice to research.

To the European colleagues who visited the 1958 ASCE Convention it should be said that Americans recognize their leadership in the creation of new ideas. It was perhaps the most important goal of that series of meetings to improve cooperation between such bodies as the International Association for Bridge and Structural Engineering, the European Committee on Concrete, and corresponding American organizations.

American structural concrete research and its effect on design are characterized by a few words: Cooperation between friends dedicated to a common cause. Structural concrete design is a combination of science, art, and experience. Let the future be built by men "touching a star" together, to give dreams shape and form; let ideas be exchanged and developed through rigorous scientific study; let then the results of research be placed into reality through the services of the civil engineering profession. It is the writer's fondest hope that this may be done by a family of nations in cooperation. For wherever a civil engineer's cradle stood, his calling is the same—the welfare of man.

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DESIGN TABLE FOR STEEL COLUMNS WITH ECCENTRIC LOADS

Richard Hugh Bigelow,[1] A. M. ASCE

ABSTRACT

This paper presents a table of bending factors for the design of eccentrically loaded steel columns with n explanation of their derivation. The factors presented differ from those of the A.I.S.C. Manual in that they allow a more precise calculation of the equivalent concentric load.

The A.I.S.C. Specification in section 12 limits the concentric and the eccentric loads to conformity with the following expression:

$$f_a/F_a + f_b/F_b \leqq 1$$

The above may be equated to the limiting value and expressed as:

$$P_h/P_a + M/20S = 1$$

where P_h = P-have = Eccentric load in kips the design must accommodate.

P_a = P-allowable = Total concentric load in kips on a given section when no bending occurs or the equivalent concentric load allowable when both a concentric load and a moment are applied.

M = Moment in inch kips.

S = Section modulus in inches cubed. It should be noted the allowable stress in bending is taken as 20 kips per sq. in. This will be discussed later.

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This may also be expressed as:

$$P_a = P_h + P_a M/20S \quad (1)$$

or

$$P_a - P_h = P_a M/20S \quad (2)$$

If M is held constant in Eq. (2) $P_a - P_h$ will vary as P_a/S varies. For cases with $L/r \leqq 120$, P_a/S may be expressed as:

$$A\left(\frac{17{,}000 - 0.485\ L^2/r^2}{Ar^2/c}\right)$$

which will reduce to

$$c\left(\frac{17{,}000 - 0.485\ L^2/r^2}{r^2}\right)$$

If L is held constant and reference is made to the A.I.S.C. manual it will be noted there is only a small variation in r and c for any one column series. A few calculations will indicate even less variation in c/r^2. The value of $P_a/20Sxx$ then is nearly constant for a particular column series and length.

For example the value of $P_a/20Sxx$ for the 12 x 12 series varies from less than 1% to a maximum of 6.8% with an average variation of 2.5%; the minimum variation being for column lengths at about the mid-height range and the maximum variation being for column lengths at the upper limit of the height range.

A solution based upon values of $P_a/20Sxx$ considered constant for any one column series would then introduce only a relatively small error in a design problem. This design procedure consists of the solution of Eq. (1) by means of the A.I.S.C. column tables pages 214 through 221 and the table below.

Example of Design Problem

Problem:

Select the lightest 12" column section for a buckling length of 22 ft. with a 300 kip load acting on the Y-Y axis 4" from the X-X axis.

Solution:

An observation of the factors in table below for 12" columns at 22 ft. length indicates the factors are in the range of 1/7 to 1/8. A quick mental calculation as follows will place the solution in the proper column series:

$$P_a \text{ (equivalent)} = 300 + \frac{300 \times 4}{8} = 450 \text{ kips}$$

It will be noted in A.I.S.C. column table this value is in the range of the 12 x 12 series. Therefore, select the value of $P_a/20S_{xx}$ from table below of 0.1465 for 12 x 12 column 22 ft. long.

Then

$$P_a\text{(equivalent)} = 300 + (300x4)\ 0.1465 = 476 \text{ kips}$$

VALUES OF $P_a/20Sxx$

VALUES OF Pa/20Sxx

Unbraced Length in Feet	NOMINAL DEPTH AND WIDTH												
	14x16	14x14.5	14x12	14x10	14x8	12x12	12x10	12x8	10x10	10x8	8x8	8x6.5	
6	.1555	.1558	.1585	.1612	.1643	.1818	.1825	.1856	.2197	.2271	.2719	.2716	
7	.1548	.1551	.1577	.1600	.1619	.1807	.1810	.1832	.2179	.2229	.2685	.2668	
8	.1542	.1544	.1565	.1584	.1588	.1795	.1796	.1801	.2160	.2186	.2646	.2596	
9	.1535	.1537	.1552	.1562	.1555	.1779	.1775	.1762	.2135	.2143	.2591	.2524	
10	.1528	.1528	.1536	.1540	.1517	.1765	.1754	.1723	.2110	.2100	.2540	.2429	
11	.1520	.1519	.1519	.1518	.1475	.1746	.1726	.1685	.2079	.2043	.2482	.2346	
12	.1511	.1507	.1503	.1491	.1432	.1727	.1697	.1638	.2047	.1986	.2412	.2236	
13	.1500	.1495	.1486	.1464	.1382	.1709	.1662	.1584	.2016	.1929	.2347	.2119	
14	.1489	.1484	.1466	.1432	.1332	.1685	.1627	.1530	.1979	.1857	.2276	.1995	
15	.1478	.1470	.1445	.1399	.1279	.1662	.1591	.1468	.1936	.1786	.2202	.1852	
16	.1464	.1456	.1421	.1367	.1221	.1634	.1556	.1406	.1896	.1700	.2119	.1708	
17	.1451	.1442	.1398	.1332	.1157	.1607	.1513	.1337	.1851	.1614	.2036		
18	.1438	.1426	.1371	.1291	.1086	.1581	.1471	.1260	.1805	.1529	.1954		
19	.1423	.1410	.1346	.1251		.1552	.1421	.1190	.1755	.1446	.1854		
20	.1407	.1391	.1318	.1207		.1524	.1372		.1706		.1755		
21	.1390	.1373	.1287	.1162		.1495	.1322		.1651				
22	.1372	.1354	.1257	.1133		.1465	.1268		.1595				
23	.1354	.1333	.1222	.1064		.1432	.1216		.1540				
24	.1334	.1312	.1188	.1015		.1400	.1159		.1481				
25	.1315	.1289	.1154			.1366			.1417				
26	.1294	.1266	.1115			.1330							
27	.1275	.1243	.1077			.1292							
28	.1255	.1218	.1039			.1254							
29	.1234	.1192	.0977			.1214							
30	.1214	.1164	.0955			.1172							
32	.1168	.1106											
34	.1126	.1049											
36	.1080	.0984											
38	.1031												
40	.0980												

Enter A.I.S.C. column table pg. 216 and find next larger value in the 12 x 12 series is 478 kips corresponding to a 12 W.F. 120 section. This is the column selected. A check will demonstrate.

$$P_h/P_a + M/20Sxx = 0.995$$

If the above problem is solved using the A.I.S.C. bending factors the value of P equivalent will fall between values in the column tables corresponding to a 12 W.F. 133 and a 12 W.F. 161 section. Knowing the bending factors give conservative results the designer would check the 12 W.F. 133 section and finding the interaction formula fraction for this section considerably less than 1 he would check the 12 W.F. 120 section and find it acceptable. In problems where the flexural stresses represent a greater percentage of the interaction formula fraction than is the case above the A.I.S.C. bending factors can give even more conservative results.

Example of Review Problem

Problem:

Determine the value of a load P acting on the Y-Y axis 2" from the X-X axis of a 10 W.F. 60 column with buckling length of 18'-0".

Solution:

Expressing $M = P_h \times e$ where e is equal to the eccentricity in inches Eq. (1) may be expressed as

$$P_h = \frac{P_a}{1 + P_a e/20Sxx}$$

In this problem then from table below $P_a/20Sxx = 0.1805$ and from A.I.S.C. column tables $P_a = 240$ kips

$$P_h = \frac{240}{1 + 0.1805 \times 2} = 176.3^k \text{ Ans.}$$

A check will demonstrate that $P_h/P_a + M/20Sxx = 0.9973$ or P_h may have a maximum value of 176.8^k. The answer above is in error by 0.3%.

If e in the problem above were increased to 10" the answer would be in error by 0.6%.

Several variations of the review problem may be solved using an approach similar to that above.

General Comments

Some variation in the value $P_a/20Sxx$ for a given column series and length was demonstrated above. The values included in the table were so derived that the required column for any equivalent concentric load calculated as shown above will correspond to either the value above or the value below the equivalent concentric load as tabulated in the A.I.S.C. column tables. In the majority of cases the required column will correspond to the tabulated value above the calculated equivalent concentric load. In some cases where the

equivalent concentric load is close to the lower tabulated value the column corresponding to this value may suffice.

The reader will note the lack of a table for bending about the Y-Y axis. Since it is more efficient and typical of present day framing to apply the loads to the flanges of the column and provide for bracing about the Y-Y axis through the web it was thought such a table would have little application.

The table is based upon the value of Ld/bt for any column being equal to or less than 600. Of the 73 columns covered by the table the six columns listed below are the only ones which have column lengths within the range of the table for which the value Ld/bt exceeds 600. It should be noted this occurs only near the maximum height range.

Column	Lu	Max. tabular length
14WF87	35.5	36
14WF61	23	24
14WF48	17	18
14WF43	15.5	18
12WF40	17.5	19
10WF33	17.5	19

The table is prepared for columns with values of L/r equal to or less than 120. The use of columns eccentrically loaded with values of L/r greater than 120 in most cases leads to a reduction of allowable bending stress as well as a continued reduction in allowable axial stress.

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LOAD DISTRIBUTION OF DIAPHRAGMS IN I-BEAM BRIDGES

Benjamin C. F. Wei,[1] A. M. ASCE

SYNOPSIS

This paper presents an analytical study of the effects of diaphragms in I-beam bridges. The structures considered in the analyses are simple-span right bridges, consisting of a continuous slab supported by five uniformly-spaced parallel beams of equal stiffness, with the beams running in the direction of traffic. The diaphragms, which may be in the form of a channel, a WF-beam, or a built-up section, are transverse framings in the structure. Several variables are studied; including (1) relative stiffness of diaphragm to that of each beam, (2) the position of diaphragms in the structure, (3) relative stiffness of beams to that of slab, (4) relative dimensions of bridge, and (5) the type and position of loading. The influence of diaphragm upon the behavior of edge beam is also included.

From the results of analyses, studies and comparisons are made to investigate the effects of different variables on the action of diaphragms in distributing the load on the bridge. The criteria of comparison is in general the maximum moment in beams produced by unit load or 4-wheel loading, with particular emphasis to the latter to simulate the actual loading condition. For one specific case, the analytical results are compared to those given for a full-size field test. Conclusions and design recommendations with regard to the effective use of diaphragms in I-beam bridges are also presented.

INTRODUCTION

1. Object and Scope of Investigation

The studies presented in this paper were undertaken to obtain a better understanding of the action of diaphragms in I-beam bridges. The I-beam bridge, consisting of a concrete roadway slab continuous over steel stringers,

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1. Prin. Designing Engr. on Structures, Hydrocarbon Research Inc., New York, N. Y.

has found widespread application for highway bridges because of its simplicity of design and ease of construction. The diaphragms, in the form of a channel, a WF-beam, or a built-up angle frames, is a transverse framing device, which is added to the structure with the following purposes; to improve the load distribution of load in the structure to the supporting beams, to facilitate erection of the span, and to stiffen, in general, the bridge so that it may act more efficiently as one integral unit under load. It is the purpose of this paper to discuss the lateral load-distribution characteristics of diaphragms in this type of structure so that a rational basis may be obtained to serve as a guide in the design of diaphragms in this most common type of highway bridges.

The data contained herein apply to simple-span right bridges consisting of a continuous slab supported by five uniformly-spaced parallel beams of equal stiffness, with diaphragms framed transversely in different positions between the beams.

The problem of the I-beam bridge with a concrete slab roadway supported on beams which are interlaced with diaphragms is one which is statically indeterminate to a high order. The exact solution to the problem is complicated and is not suitable for practical use. A numerical distribution procedure for the analysis of slabs continuous over flexible beams by Newmark[1] furnishes a convenient tool for analyzing the structure and is followed in this investigation. With this basic procedure and a technique developed for superposing the action of diaphragms to the flexible beams under consideration, it was possible to undertake a large number of analyses and, thus, to investigate the effects of the following variables on the action of diaphragms in the structure; (1) relative stiffness of diaphragm to that of each beam, (2) the position of diaphragms in the structure, (3) relative stiffness of beams to that of slab, (4) relative dimensions of the bridge, and (5) the type and position of loading. For one case the analytical results were used to compare with those derived from field test of a full size bridge.

The bridges considered have beams in the direction of traffic and a ratio of beam spacing to span equal to o.1. The flexibility of the beams is taken into account in the analyses. In general, the maximum moments are determined in the beams for different positions of a concentrated load on the structure. Bridges with diaphragms of various stiffnesses and framed at midspan, each third-point, each quarter-point, and midspan plus quarter points are considered. Influence coefficients for moment are obtained for a group of these structures of various proportions and for different relative stiffnesses of beams and slab. From the influence values, moments in the beams are determined for standard highway truck loads for spacings of beams between the limits of 5 to 8 ft. On the basis of moments in beams due to unit load and truck loading, studies of the results of analyses are made to determine the influence of the variables as affecting the load-distributing behavior of the diaphragms in the structure. Special attentions have been directed to explore questions often raised by designing engineers; such as, "How effective is a relatively flexible diaphragm?", "Are the effectiveness of diaphragm proportional to its stiffness?", "What is the best position of a diaphragm?", and "Are closed-spaced diaphragms necessarily more effective than a single one?", etc. The effect of diaphragms on the behavior of the edge beam is also studied.

2. Basic Assumptions

The distribution procedure used in the analysis is based on the ordinary theory of flexure for plates. The following assumptions are involved in the solution of the problem:

(1) The material in the slab is homogeneous, elastic, isotropic, and of constant thickness.
(2) The resultant of the normal stresses acting on any cross-section of the slab is a pure couple.
(3) Flexural strains vary linearly through the depth of the slab.
(4) The beams exert only vertical forces on the slab; there is no shear between the top flange of the beams and the bottom of the slab.
(5) The reaction of the beam acts on the slab along a line and is not distributed over a finite width. The torsional restraint offered by the I-beams is not taken into account in the analysis.
(6) A beam and the slab directly over it deflect the same amounts and in the same shape; that is, there is no separation between the slab and the beam.
(7) The connections of the diaphragms to the beams are fully effective. In other words, the right angle relationship between the web of the beam and a line through the mid-height of the diaphragm will be kept after deformation. This condition will permit rotation of beams as shown in Fig. 1.
(8) The top of the diaphragm does not touch the bottom of the slab under any loading condition.
(9) Both the slab and the beams are simply supported at the ends of the span.

In addition, the value of Poisson's ratio for concrete is assumed to be zero in all the calculations.

3. Notation

The following notation is used throughout the paper. The longitudinal direction is always taken as the direction of the beams.

$\underline{a}$ = span of bridge, center to center of supports.

$\underline{b}$ = transverse spacing of beams.

$\underline{L}$ = Length of the diaphragm when it is considered as one cross beam, equal to 4 b for the bridges analyzed.

$\underline{h}$ = total depth of slab.

$\underline{I}$ = moment of inertia per unit of width of the cross-section of the slab; for a homogeneous material, $\underline{I} = \underline{h}^3/12$.

$\underline{E}$ = modulus of elasticity of the material in the slab.

μ = Poisson's ratio for the material in the slab.

$\underline{N} = \frac{EI}{1 - \mu^2}$ = measure of stiffness of an element of the slab.

E_b = modulus of elasticity of the material in a beam.

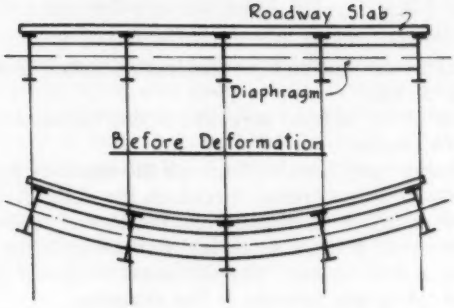
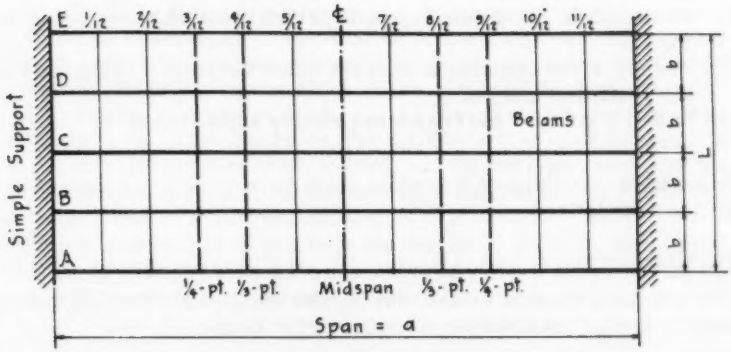
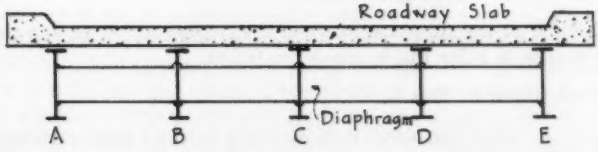


FIG. 1 ROTATION OF BEAMS AFTER DEFORMATION



Plan of Bridge



Transverse Section of Bridge

FIG. 2 PLAN AND SECTION OF I-BEAM BRIDGE

E_d = modulus of elasticity of the material in a diaphragm.

I_b = moment of inertia of the cross-section of a beam (or transformed moment of inertia of a T-beam).

I_d = moment of inertia of the cross-section of a diaphragm.

$\underline{H} = \frac{E_b I_b}{a N}$ = a dimensionless coefficient which is a measure of the stiffness of a beam relative to that of the slab.

$\underline{r} = \frac{E_d I_d}{E_b I_b}$ = a dimensionless coefficient which is a measure of the stiffness of a diaphragm relative to that of a beam.

$\underline{P}$ = concentrated load applied to the slab.

Δ = vertical deflection of beams, positive downward.

δ = vertical deflection of diaphragms, positive downward.

$\underline{R}$ = reactions offered by the diaphragms to the beams in the bridge.

$\underline{M}$ = bending moment in a beam or in a T-beam, positive when producing compression in the top fibers.

Method of Analysis

4. Fundamental Concepts of Load Distribution in I-Beam Bridges

Fig. 2 shows a typical plan and section of the bridge analyzed. The diaphragms have been placed at one of the following positions; namely, midspan, each third-point, each quarter-point, and midspan plus quarter points. To avoid confusion, it must be understood that the designation "each quarter-point" used throughout the paper is meant only those two positions at one-quarter of the span from the simply supported ends, not including the center point. The three-point case is designated as the "midspan plus quarter points".

Consider first an I-beam bridge without diaphragms. When a load is applied to the bridge, it is distributed by the slab to the beams and ultimately by the beams to the abutments. Obviously, the beams act as principal load-carrying elements in an I-beam bridge. Since the loads are distributed to the beams through the slab, the proportion of load each beam carries depends to a great extent on the stiffness of slab. The relation between the stiffness of beam and slab is expressed by a dimensionless coefficient:

$$H = \frac{E_b I_b}{a N}$$

where E_b = modulus of elasticity of the material in a beam,

I_b = moment of inertia of cross-section of beam,

$N = \frac{EI}{1 - \mu^2}$,

E = modulus of elasticity of the material in the slab,

I = moment of inertia per unit of width of the cross-section of the slab, and

μ = Poisson's ratio, taken as zero in the data given in this paper.

For a bridge having a proportion of beam spacing to span equal to 0.1 and the particular value of H = 5, the approximate load distribution on beams may be shown as in Fig. 3. It is interesting to note that when the load is applied over beam B, the slab tends to spread the load to the adjacent beams, thus reducing the burden of beam B. The load thus transferred takes the form approximately of a sine curve of distributed load as shown in the figure. The total distributed load is indicated on each curve.

The distribution illustrated in Fig. 3 is true only for the bridge with H = 5. A different relation between the stiffnesses of the beam and slab would give different results. From the relation defined for H, it is evident that a low value of H corresponds to a stiff slab and a high value to a flexible slab. The effect of slab stiffness on the moment in the beams for a single load on the bridge is indicated in Fig. 4, which shows the influence lines for moments at midspan of beam C, the center beam in a 5-beam bridge in which b/a = 0.1. It may be noted from the figure that for the curve $H = \infty$, which corresponds to a slab of zero stiffness, the static moment due to a load at C is carried entirely by beam C and is equal to 0.25 Pa, the other beams all having zero moment. On the other hand, if H is reduced to 0.5, corresponding to a relatively stiff slab, the moment in beam C for a load at C becomes 0.064 Pa, about 25 per cent as large as the previous value. The moments in the adjacent beams for the latter case are almost equal to that in the center beam. This fact demonstrates the importance of the slab thickness in distributing the load to the beams. However, a relatively thick slab causes heavy dead load on the structure and furthermore involves excessive cost of concrete. Thus, in some instances where the roadway slab is thin, and therefore flexible, a transverse framing device such as diaphragms may be introduced to assist in the distribution of load. From the curves shown in Fig. 4, it is conceivable that if all other conditions remain the same, diaphragms will prove to be more effective in a structure with a high H, having a relatively flexible slab, than in one with a low H, having a relatively stiff slab.

The major difference between the slab and diaphragm as a means of distributing the load lies in the nature of the loading transferred to the beams. For a diaphragm, the loads are transferred in the form of concentrated loads applied at the points where the diaphragms intersect the beams. These loads, instead of the approximate sine curve of distributed loads shown in Fig. 3 for the slab, constitute the elements to reduce the burden of the loaded beam. In this paper, all the bridges treated have a roadway slab. The addition of diaphragms to these structures will cause a combination of the action of the approximate sine curve of distributed loads and the concentrated loads.

5. General Description of Principle of Analysis

The basic principle of the solution of the problem is to replace the action of the diaphragms in the structure by a set of equivalent forces acting at the points of intersection of the diaphragms and the beams. The problem is then reduced to one of finding moments in a bridge without diaphragms but subjected to additional forces. The analytical solution of a large number of simple-span right bridges, consisting of a slab supported by five identical parallel beams but without diaphragms, has previously been made by using the distribution procedure described in Bulletin 304,(1) and numerous influence tables for moment and deflection in beams have been reported.(2) These influence

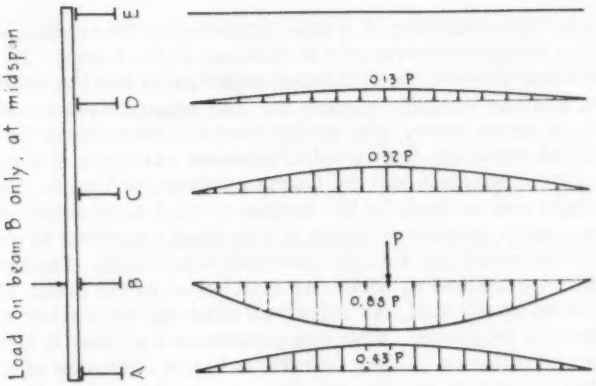


FIG.3 ACTION OF SLAB IN DISTRIBUTING CONCENTRATED LOAD (b/a = 0.1 and H=5)

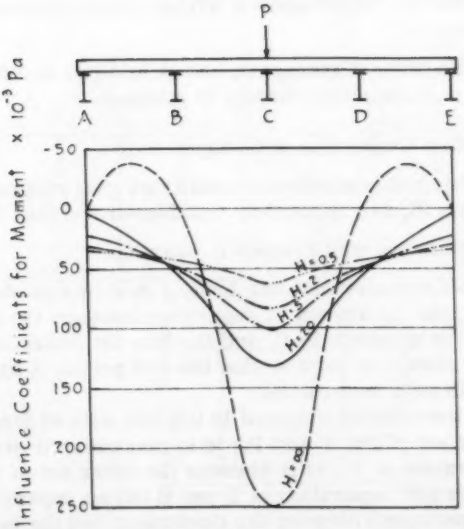


FIG.4 EFFECT OF SLAB STIFFNESS ON THE MOMENTS IN BEAMS, FOR A SINGLE LOAD IN THE BRIDGE

tables have been used for the calculation of influence coefficients for moment in beams in the same structure with the addition of diaphragms. It must be noted that the values of moments and deflections for the bridges without diaphragms computed as described above agree closely with those measured in tests.(3)

Consider a bridge consisting of a slab supported by five identical I-beams with diaphragms framed transversely at midspan of the bridge. By assuming that the connections between the individual diaphragms and the beams are fully effective, one can virtually replace the four separate pieces of diaphragm by one continuous cross beam. The bridge may now be analyzed by removing the diaphragm and replacing it by a set of unknown reactions at the points of intersection of the diaphragm and the beams as shown in Fig. 5. This set of unknown reactions can be found by the method of consistent deformation. In order to solve a large number of cases, it was found expedient to break the load into its symmetrical and antisymmetrical components. The two components are then superposed to obtain the final effect of the load. The advantage of splitting up the load lies chiefly in reducing the number of simultaneous equations to be solved. Also this procedure was used to treat different values of r and different loading conditions in one operation of calculation. An outline of the method of analysis for three cases differing in the arrangement of diaphragms is presented to illustrate the procedure:

Case 1. Bridge with one diaphragm at midspan only.

Case 2. Bridge with two diaphragms at symmetrical positions with respect to midspan.

Case 3. Bridge with three diaphragms, one at midspan and two at symmetrical positions with respect to midspan.

Case 1. Bridge with One Diaphragm at Midspan Only

The symmetrical and antisymmetrical conditions with respect to the center beam, beam C (see Fig. 2), are separately considered and then combined.

(A) Symmetrical condition with respect to beam C.

(a) Using the two symmetrical unit loading systems as shown in Fig. 6 (a) and (b), find the amount of separation between the diaphragm and the beams at points B, C, and D. The net deflection for the two free bodies should be used so that the end points, A and E, are kept together with zero separation.

(b) Find, from the relation obtained in (a), two sets of forces $P_{B,\,D}$ and P_C; one set of $P_{B,\,D}$ and P_C is to produce unit separation at C but no separation at B and D whereas the other set of $P_{B,\,D}$ and P_C is to produce unit separation at B and D but no separation at C.

(c) Find the separations between the diaphragm and the beams at B, C, and D, relative to a horizontal line through A and E, due to the symmetrical component of the external load acting on the structure. The antisymmetrical component of external loading will be taken care of in step (c) of (B).

(d) From the coefficients in (b) and the separations found in (c), find the forces that must be present in order to make the separations equal to zero. Forces will also be present at the end points to

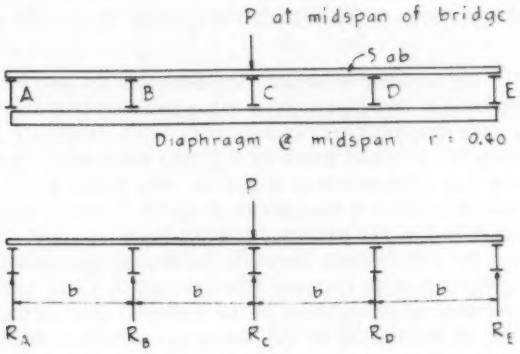


FIG.5 DIAPHRAGM REPLACED BY CONCENTRATED LOADS

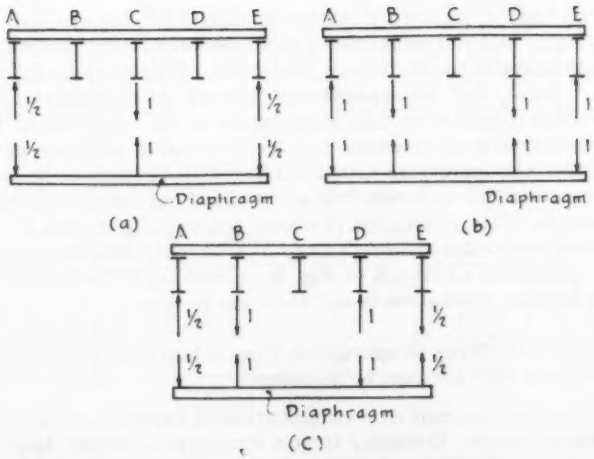


FIG.6 SYMMETRICAL AND ANTI-SYMMETRICAL UNIT LOADING SYSTEMS

satisfy statics. These forces or reactions represent the restraint to deflection offered by the diaphragm to the bridge.

(B) Antisymmetrical condition with respect to beam C.—The treatment of this part is similar to that in (A) except that in this case, because of antisymmetry, there would be no separation at point C as well as at points A and E.

(a) Find the amount of separation between the diaphragm and the beams at points B and D, relative to an inclined line through A-C-E, under an anti-symmetrical system of loading as shown in Fig. 6(c).
(b) From the relation found in (a), find the force $P_{B,\ D}$ which will produce unit separation at B and D. The force P_C is not necessary since the relative separation at point C due to antisymmetrical component of the external load is equal to zero.
(c) Find the separations between the diaphragm and the beams at B and D, relative to an inclined line through A-C-E, due to the antisymmetrical component of the external load acting on the structure.
(d) Find, as in step (d) of (A), using the relation from (b) and (c), the forces that must be present to make the separations equal to zero.

(C) Combine the reactions obtained in (d) for both the symmetrical and antisymmetrical conditions. The bridge may then be analyzed as one without diaphragms but with additional forces acting along the diaphragm line on beams A, B, C, D, and E.

Case 2. Bridge with Two Diaphragms at Symmetrical Positions with Respect to Midspan

The method used to analyze this case is similar to the one outlined in detail for Case 1. It must be borne in mind, however, that since there are now two diaphragms in the structure, one must, in steps (A) - (a) and (B) - (a) mentioned above, find the separations between the diaphragms and beams due to the loading systems on both diaphragms at the same time. Furthermore, if the external load is acting at a location not at midspan of the bridge, the symmetrical and antisymmetrical loading with respect to beam C must further be resolved into symmetrical and anti-symmetrical loadings with respect to midspan. This resolution is shown graphically in Fig. 7.

The combination as described in Case 1 (C) should then be composed of four loading conditions as shown in Fig. 7. However, if the load is applied at midspan, the further resolution is not necessary.

Case 3. Bridge with Three Diaphragms, One at Midspan and Two at Symmetrical Positions with Respect to Midspan

This case can be analyzed as a combination of Cases 1 and 2 by using the method of superposition. Consider first a structure with two diaphragms at symmetrical positions with respect to midspan as the basic structure. Use the influence coefficients for moment and deflection found for this basic structure and analyze the original structure as if there is only one diaphragm at midspan. A similar procedure may be followed for a bridge with more than three diaphragms.

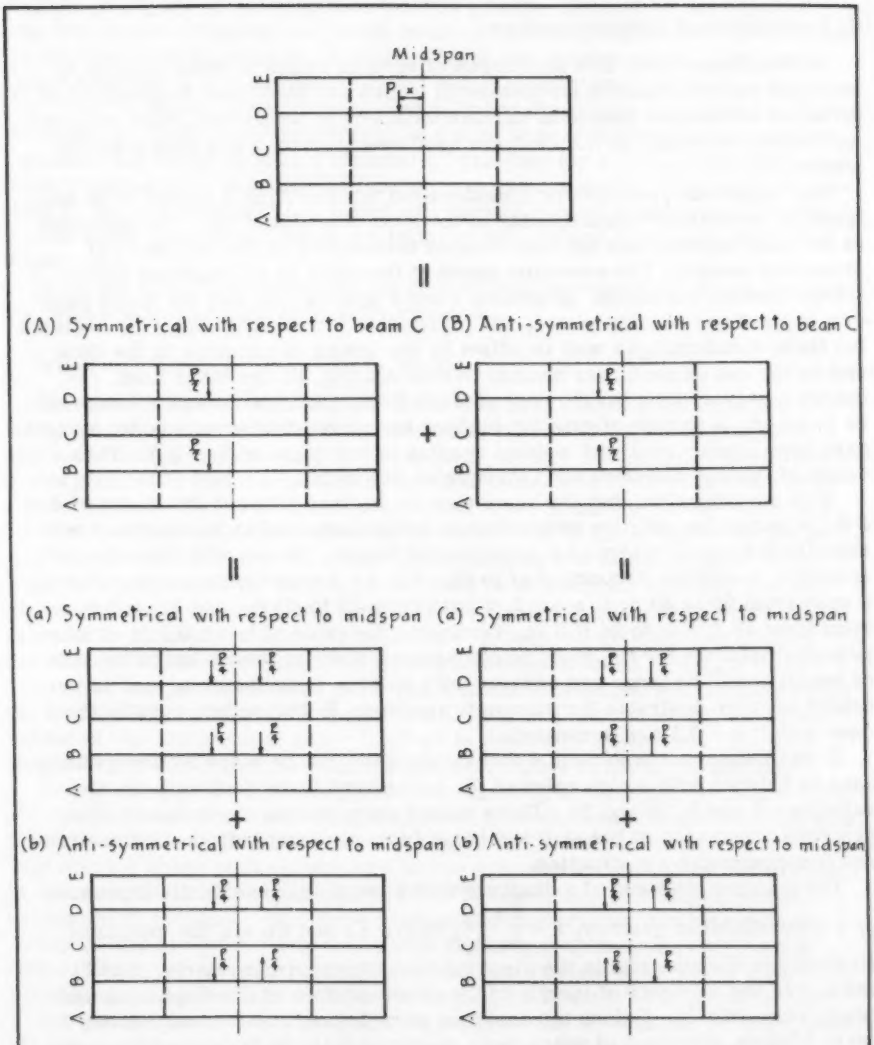

Midspan
P
x
A B C D E
(A) Symmetrical with respect to beam C
(B) Anti-symmetrical with respect to beam C
P/2
(a) Symmetrical with respect to midspan
(a) Symmetrical with respect to midspan
P/4
(b) Anti-symmetrical with respect to midspan
(b) Anti-symmetrical with respect to midspan

FIG.7 RESOLUTION OF FORCE INTO COMPONENTS IN BRIDGE WITH TWO SYMMETRICALLY-SPACED DIAPHRAGMS

Results of Analyses

6. Description of Bridges Analyzed

All together eighty-five structures of various ratios of beam spacing to span and various relative stiffnesses of beams and slab, with diaphragms of different stiffnesses spaced at various locations in the bridge, were analyzed and complete results of the analyses have been reported in a thesis by the writer.[4]

One important preliminary consideration for selecting a bridge to be analyzed is the ratio of beam spacing to the span of bridge (b/a). The selection of the beam spacing has not been treated extensively in the literature of structural design. The economic phase of the study is complicated and involves several variables. In general closer spacing will call for more rockers, rollers and bearing plates, and more steel for the diaphragm connections, but these disadvantages may be offset by the saving of concrete in the deck and by the use of shallower beams. Wider spacing, on the other hand, requires a heavier slab but the cost of steel fabrication and concrete forms will be reduced. A survey of existing bridges has shown that most I-beam bridges have been constructed with a beam spacing in the range of 5 to 8 ft. This range of spacing has been used throughout this paper.

With consideration that the beam spacing is fixed between the limits of 5 to 8 ft., a particular relative proportion of bridge assumed in the analyses would actually indicate a bridge of a certain span length. Hence, with these beam spacings, a relative proportion of bridge $b/a = 0.1$ represents a bridge having a span from 50 to 80 ft; $b/a = 0.2$, a span from 25 to 40 ft., and $b/a = 0.3$, a span from 16 ft 8 in to 26 ft 8 in. Obviously the value of $b/a = 0.1$ is of more practical interest for the study of diaphragms since in longer spans the size of beams would be large and consequently an even distribution of load to the beams is more desirable for economic reasons. In this paper, results for case with $b/a = 0.1$ are presented.

It was mentioned in Section 4 that diaphragms can be more advantageously used in bridges with a high value of $\underline{H}$. In the structures analyzed, the $\underline{H}$ values used are 5, 10 and 20. These values were chosen to represent most practical structures of the slab and beam type, covering both the composite and non-composite construction.

The relative stiffness of a diaphragm to a beam is conveniently expressed by a dimensionless quantity, $r = \frac{E_d I_d}{E_b I_b}$, where E_d and E_b are the moduli of elasticity of the material in the diaphragm and beam, respectively, and I_d and I_b are the moments of inertia of the cross-section of the diaphragm and beam, respectively. Before the analyses were begun, a survey of existing I-beam bridges, the plans of which were collected from 30 States and Ontario, Canada, was made. Results of the survey indicated that the most common relative stiffness of steel diaphragm (r) is approximately equal to 0.05. In the data reported in this paper, values of $\underline{r}$ equal to 0, 0.05, 0.15, 0.40, and infinity were, in general, used. These values cover the extreme cases of no diaphragms and of infinitely stiff diaphragms, as well as the practical range from relatively flexible diaphragms to relatively stiff ones. The value of $\underline{r}$ equal to infinity was introduced into the analysis to serve as basis for comparison in cases of extremely stiff diaphragms.

The diaphragms were generally placed at midspan, each third-point, each quarter-point, and at midspan plus quarter points. Because of the relatively long and tedious computational work involved in evaluating moment coefficients for the case with diaphragms at both midspan and quarter points, this case was considered in the analyses only for one value of $\underline{r}$ equal to 0.15 and for b/a = 0.1. It is felt that by considering only one diaphragm stiffness for this arrangement useful conclusions may be drawn to permit some generalization regarding the action of such a structure. Analysis for a bridge with more than three diaphragms is too time consuming and was not made. In general the results of the investigations presented will furnish enough information to allow a reasonably good prediction of the behavior of a bridge with more than three diaphragms.

In all the structures analyzed, the connections between the diaphragms and the beams were assumed to be fully effective.

7. Influence Values for Moments in Beams

For each bridge considered, the basic step is to obtain a set of numerical values of influence coefficients for moments at various points in beams for a unit load moving transversely on the bridge. Before the analyses were begun, it was planned to find the maximum moment in the beam. However, the complexity of the action of structure under load made it hard to predict at which location longitudinally the maximum moment in the beam would occur for a single load moving along the beam. For example, if a load is applied away from midspan, would the maximum moment be at midspan or under the load? Furthermore, if the maximum occurs under the load, would it occur at midspan, or at some other point on the beam? A considerable amount of time can be saved in computation once a generalization is found. Thus, a large number of computations were made for moment at midspan and moment under the load for the load applied at different locations along the beam; the result indicated that the moment under the load is generally the larger. These moments directly under the load are therefore used in the investigation.

Longitudinally, the load is applied successively at midspan, 5/12 a, 4/12 a, and 3/12 a, these positions being measured from the end of the bridge. It was found from the results of computations made for the different positions of the load that for cases with diaphragms at two symmetrical positions with respect to midspan, the maximum moments in all beams occur at midspan of the bridge under the load. For these positions of diaphragms, the moments at midspan only are used. For cases with diaphragm at midspan only, and at both midspan and quarter points, the maximum moment point shifts and, consequently, moments at midspan, 5/12 a, 4/12 a and 3/12 a are derived. The value and position of the maximum moment can be obtained by interpolating from curves drawn with the known moment as ordinates and their location as abscissas.

Fig. 8 shows a set of influence lines for edge, intermediate and center beams for a bridge with b/a = 0.1, H = 10 and diaphragms at quarter-points. The values of $\underline{r}$ considered are 0, 0.05, 0.15, 0.40, and infinity. It is somewhat difficult to draw accurately curves of this type with only five points defined, particularly when a large horizontal scale is being used. However, the plotting is greatly facilitated by following the general shape of the curve for r = 0, for which the values of moment at points between the beams are known.(2)

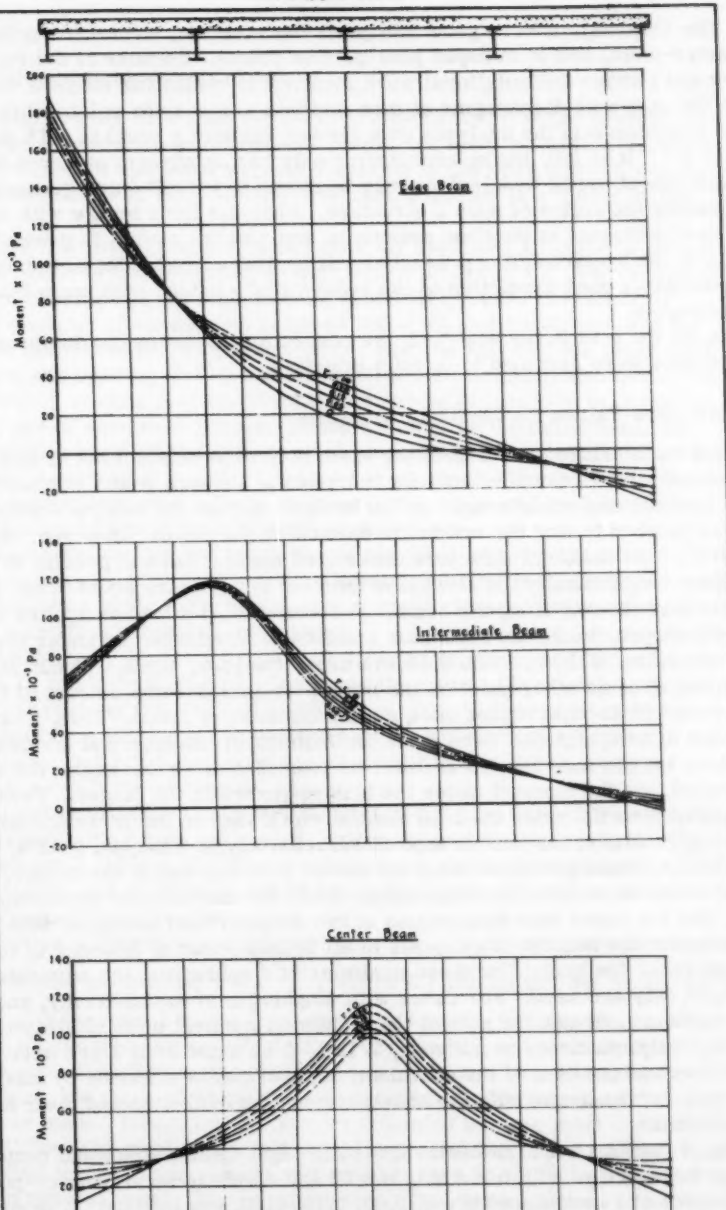


FIG. 8 INFLUENCE LINES FOR MOMENTS IN BEAMS AT MIDSPAN FOR A LOAD MOVING ACROSS THE BRIDGE AT MIDSPAN ($B/_0$ = 0.1, H=10, and Diaphragms at Each Quarter-Point)

From the shape of the influence curves for moment in the edge beam as shown in Fig. 8, it may be seen that the addition of more beams on the far side of the slab would generally cause little change in the moment carried by this beam. The influence line for an intermediate beam would also be changed only slightly by the addition of more beams on the far side. The addition of more beams on either or both sides of the bridge might bring about some change in the influence lines for moments in the center beam, but in general it is felt that the results obtained for five beams may be used for structures having either four beams or a larger number of beams than five.

8. Moments in Beams for Standard Truck Loads

The standard truck loading for which moments are given is that specified in the AASHO Standard Specification for Highway Bridges. The standard H truck has the wheels of each axle spaced 6 ft. apart, with front and rear axles spaced 14 ft. apart. Each of the rear wheels carries a weight of four-tenths of the total weight of the truck and each of the front wheels carries one-tenth of the total weight of the truck, or one fourth of the rear wheel weight. The weight of the truck in tons is designated by a numeral following H, as H-20. The rear wheel load P, in terms of which the maximum moment coefficients in this paper are stated, is the weight on a rear wheel increased by the impact factor. The front wheel is always taken as P/4.

Each truck is considered to occupy the central part of a 10-ft. traffic lane; therefore, the minimum distance between the center of a wheel and the face of a curb is taken at 2 ft., and the minimum distance between the centers of wheels of trucks in adjacent lanes is taken as 4 ft.

Beam spacings of 5, 6, 7, and 8 ft. are considered. These spacings cover the range of dimensions of most I-beam bridges. For the purpose of spacing the loads on the bridge, the face of the curb has been considered to lie directly over the centerline of the edge beam. The loads are then applied a minimum of 2 ft. from the edge beam and the maximum value of moment for each beam is obtained by placing these loads at the highest ordinates of the respective influence lines. Results for this case of loading are given for beam spacings of 6, 7, and 8 ft. Unless noted otherwise this loading condition is used in plotting all graphs presented in this paper. For narrow beam spacings of 5 and 6 ft., results are given for the additional case of the outer wheel coming directly over the edge beam to produce a maximum value. This condition of loading may occur in actual circumstances since in a narrow bridge the outer wheel of the truck may come nearer to the curb; in case the face of the curb of the bridge is projected 2 ft. outside the edge beam, this position of the loads still satisfies the AASHO Specification. This special wheel load condition will be designated as 'Edge Loading'.

For the proportion of bridges discussed in this paper, two lanes of loading generally produce the maximum moments in the beams. This condition of loading is possible when one truck passes another in the adjacent lane. For purpose of simplification, only one truck in each lane and only the rear wheel loads of that truck are considered in the paper.

As mentioned before, the shape of the influence line is approximated in the regions between the beams. The data for the truck loading, which are the results of computation based on the influence curves, should not therefore be regarded as exact. In general, the values computed from the influence lines are plotted against the relative diaphragm stiffnesses. A smooth curve joining

the computed points assures consistent value for different relative diaphragm stiffnesses.

9. Comparison of Analytical and Test Results

In May 1957 a paper in ASCE Journal was published[5] describing a test of a full size I-beam bridge. Part of the test program was to study the effectiveness of the diaphragms in lateral distribution of loads in such a bridge. The bridge, representing one typical structure in the City of Houston Memorial Drive Project, is a 3-span continuous I-beam structure with the span lengths of 72-108-72 ft. The side and the center spans consist partly of a constant 33WF141 of 45 ft. and 54 ft., respectively, joined over the intermediate pier by a 54 ft. haunched section made of two ST 16WF70.5 with a filler plate. The diaphragm studied is of a trussed type placed at midspan of the 54 ft. section of 33WF. Each half of the bridge consists of five longitudinal beams as described above, running in the direction of traffic and spaced at 7'-6" apart. The concrete slab has an average thickness of 6-1/4".

When the positive moment section of the center span of the test bridge is considered as the simple span length, the span would roughly correspond to an analytical bridge with the proportion of b/a = 0.1, H = 5 and a relative stiffness of diaphragm r = 0.40. Here, for conservative purposes, the value of H is computed on the basis of a non-composite section. The results of comparison were presented in the discussion of the paper.[6] Fig. 9 showing a comparison of the percentage distribution of total moment to beams in the bridge is reproduced here. Three curves are indicated; the first one given by the test, the second one from the results of this analysis and the third one for the same bridge with the diaphragms removed. The third curve was shown to serve as a limiting case where the load distribution is entirely due to concrete slab. It can be seen from the figure that with the addition of diaphragm, the total reduction of moment in the center beam under loading II is much more than that in the edge beam under loading I. In general, the comparison seems to indicate a reasonably good agreement between the test and the theoretical results.

Discussion of Results

10. Relative Stiffness of Diaphragm to Beams

One of the problems that confronts most highway bridge engineers in connection with the design of diaphragms is how stiff a diaphragm should be used in an I-beam bridge of certain proportions. Would the diaphragms always be effective, or would the effectiveness be directly proportional to their stiffness? Would the diaphragm behave in a similar manner under unit load as under truck loading? How much reduction of moment can be achieved by introducing a relatively flexible diaphragm in an I-beam bridge with a span say from 50 to 80 ft.? It is one of the purposes of this paper to find answers to these questions. As mentioned before, the study has considered a range of stiffness of diaphragm, corresponding to relatively flexible to relatively stiff ones as built in actual practice. Furthermore diaphragms of infinite stiffness are introduced in each case to serve as a limiting condition for the effect of extremely stiff diaphragms.

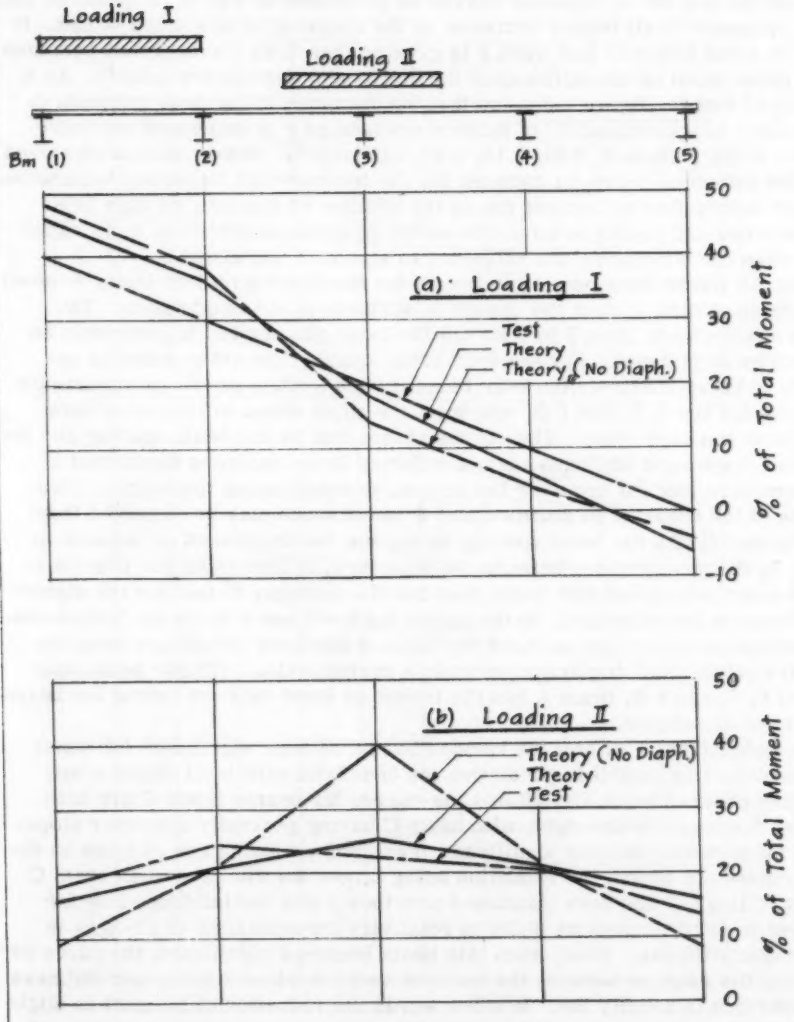


FIG. 9. THEORETICAL AND TEST RESULTS OF DISTRIBUTION OF TOTAL MOMENT TO BEAMS

ST 5

Under unit load, the effect of increasing the stiffness of a diaphragm can be seen from a set of influence curves as presented in Fig. 8. In general, the peak moments in all beams decrease as the diaphragms are made stiffer. It may be noted however that when r is greater than 0.40, the decrease becomes less pronounced as the stiffness of the diaphragm approaches infinity. As a matter of fact the figure indicates that the decrease in the peak moments in the beams is approximately of same magnitude as r is increased one value higher in the pattern 0, 0.05, 0.15, 0.40, and infinity. It may also be observed that the influence curve for moment for the intermediate beam is comparatively less susceptible to changes due to the addition of diaphragms than those for the edge and center beams. These two phenomena are found to be significant when the structures are subjected to standard truck loads.

Fig. 10 shows the moments in beams for the same structure under 4-wheel loading as plotted against the change in stiffness of the diaphragms. The beam spacings are from 5 to 8 ft. and the loads are applied transversely on the bridge at midspan. For the 5-ft. beam spacing, the outer wheel is assumed to be applied over the edge beam if such loading produces a maximum, whereas for the 6, 7, and 8 ft. spacings, the outer wheel is located at least 2 ft. from the edge beam. The curves reveal that as the beam spacing and the relative diaphragm stiffness vary, a different beam becomes significant in the structure since it acquires the largest moment among the beams. Two trends of the behavior of beams under 4-wheel loads may be observed from the figure: (1) As the beam spacing increases, the increment of moment in beam B, the intermediate beam in the structure, is always larger than those in the other beams and this beam thus has the tendency to become the significant beam in the structure. In the figure for b = 7 and 8 ft., beam B does have the largest moment; and for b = 6 ft., beam B becomes significant when the relative stiffness of diaphragm exceeds a certain value. (2) For beam spacings of 6, 7, and 8 ft. beam A has the lesser or least moment among the beams and is not significant.

An understanding of how the various beams become significant for different conditions is helpful in predicting the beneficial effects of diaphragms. One may observe from Fig. 10 that the curves for beams B and C are both sloping downward to the right, with beam C having generally a steeper slope. When these beams become significant, maximum moments are reduced as the diaphragms are added, the reduction being larger for the case when beam C is controlling. It has been mentioned previously that the influence line for moment in the intermediate beam is relatively irresponsible to changes in diaphragm stiffness. Thus, when this beam becomes significant, the curve expressing the relation between the moment under 4-wheel loading and stiffness of diaphragm is usually flat. In other words the reduction of moment is slight even with the addition of a stiff diaphragm. However, the characteristics of the curve for beam A are such that the moment increases as the diaphragm stiffness is increased. This unusual phenomenon for edge beam will be fully discussed in Section 13 'Behavior of Edge Beams'.

Consider again the curves in Fig. 10. One consistent phenomenon may be observed for all beam spacings; that is, the curve for all the beams flatten out almost to a horizontal line when it passes the value of r = 0.40. This indicates that only a negligible change of moment can be obtained by using a diaphragm stiffer than 0.40. In fact this characteristic was found to be present in all of the b/a = 0.1 cases analyzed with diaphragms at midspan, at each quarter-point, and at each third-point. This phenomenon is due chiefly

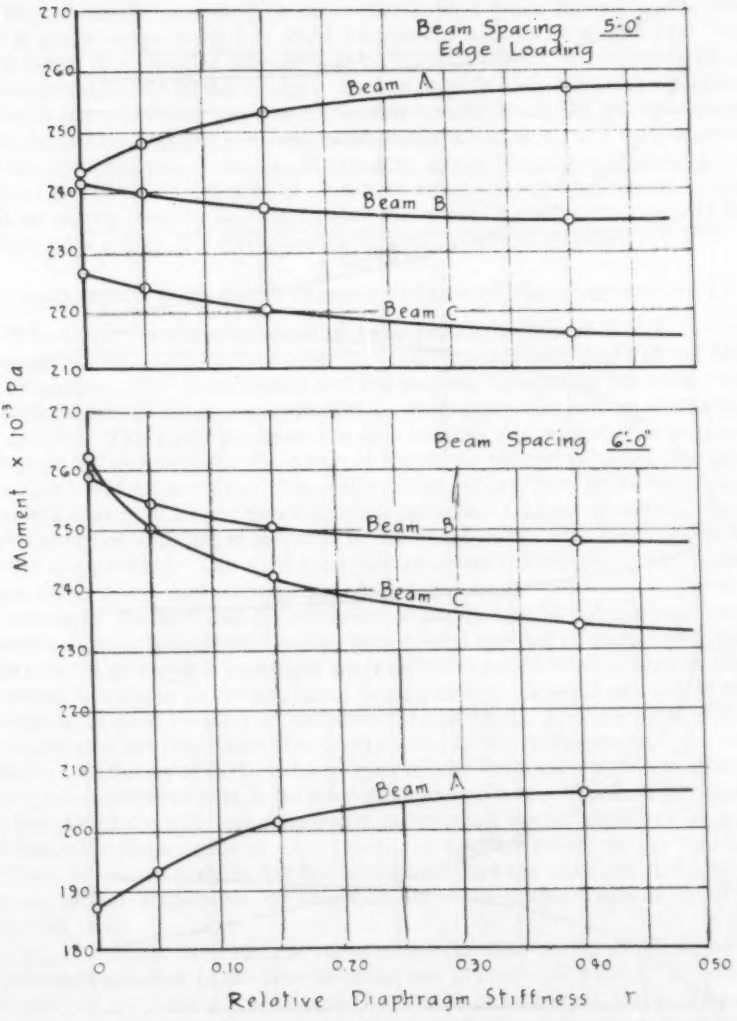


FIG. 10 RELATION BETWEEN RELATIVE DIAPHRAGM STIFFNESS AND MAXIMUM MOMENT DUE TO 4-WHEEL LOADING ($^b/_a$ = 0.1, H=10, and Diaphragms at Each Quarter-Point)

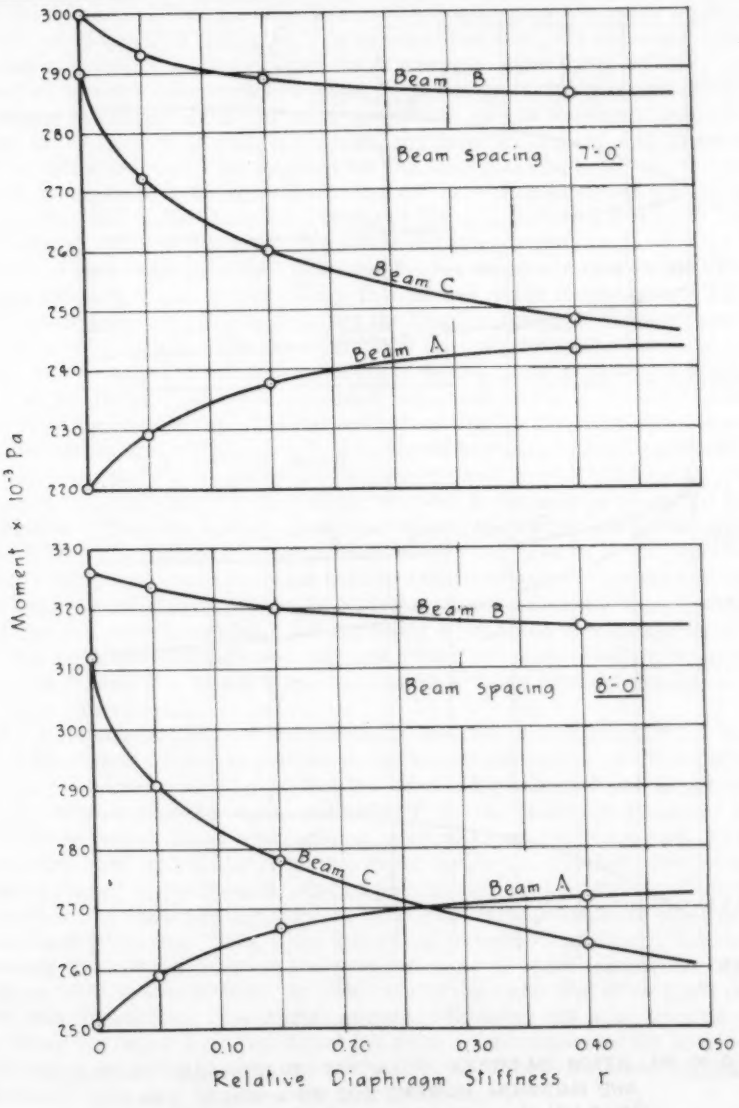

Moment × 10⁻³ Pa
300
290
280
270
260
250
240
230
220
Beam B
Beam Spacing 7'-0"
Beam C
Beam A
330
320
310
300
290
280
270
260
250
Beam B
Beam Spacing 8'-0"
Beam C
Beam A
0
0.10
0.20
0.30
0.40
0.50
Relative Diaphragm Stiffness r

FIG.10 (CONT'D)

to the proximity of the influence lines for r = 0.40 and r = infinity. In Section 12 it will be shown that the same characteristic exists for different stiffnesses of beams and slab.

The statement may then be made that in an I-beam bridge having b/a = 0.1 and H in the range from 5 to 20, a diaphragm with EI of 40 per cent that of each beam is almost as effective as an infinitely stiff one in reducing the maximum moment in the bridge. As a matter of fact, with due considerations given to the relatively greater reduction in moment in the range of small diaphragm stiffnesses and the impracticability of using a very stiff diaphragm in actual construction, a flexible diaphragm with a relative stiffness in the range of 5 to 10 per cent of that of the beam is recommended for use in design. It will be shown later in Section 12 that the lower limit (5%) is favored for bridge with a high H corresponding to composite construction.

11. Comparison Study for Different Positions of Diaphragms

It has been mentioned before that the redistribution of load to the beams through the diaphragms takes the form of concentrated loads at the points of intersection of the diaphragms and the beams. Obviously the most effective positions for the diaphragms should be at or near the section where the load is applied. The study presented in this section was undertaken to investigate the position at which the diaphragms should be placed in order to distribute the load most effectively. The criterion of comparison is the maximum moment that may be produced under a specified loading condition. Relative stiffnesses of diaphragm to beam in the range of r = 0 to infinity are included in the comparison. The data from the analyses for diaphragms at midspan, each third-point, and each quarter-point are used.

Owing to the fact that the stiffness of diaphragm is not proportional to its weight, comparison of structures with a total amount of diaphragm stiffness distributed at various positions such as midspan or third-points would not provide a criteria in the practical sense; hence, an equal amount of diaphragm stiffness in each location of the structure is used. For example, if the structure is provided with two diaphragms at the third-points, each with a relative stiffness of 0.10, it is compared to a structure with a single diaphragm at midspan with a relative stiffness of 0.10. The cost of diaphragms in the structure with one diaphragm at midspan would therefore be only half of that with diaphragms at each third- or quarter-point. In the figures that follow, the value r shown on the curves denotes the relative stiffness of the single center diaphragm, or the stiffness of each diaphragm at third or quarter point.

Fig. 11 shows the relation between the position of the diaphragms and the maximum moment in the loaded beam due to unit load moving across the bridge for the case b/a = 0.1 and H = 10. Note that when there is no diaphragm, the maximum moment in the structure is constant and is represented by a horizontal line. Each curve is determined by three points, representing diaphragms at midspan, each third-point, and each quarter-point. For the structure with diaphragms at each third- or quarter-point, the maximum moment occurs at midspan of the bridge, whereas the maximum moment for diaphragm at midspan may not be at midspan and the moments plotted in Fig. 11 were obtained by interpolating between computed moments at midspan, and at 5/12, 4/12, and 3/12 of the span length from the end of the bridge. In Fig. 11, all the curves with diaphragms are found below the straight line,

FIG.11 RELATION BETWEEN POSITION OF DIAPHRAGM AND MAXIUM MOMENT IN LOADED BEAM DUE TO UNIT LOAD (b_0 =0.1 AND H=10)

r = 0. This fact indicates that under unit load the moments in all three beams including the edge beam are reduced with the addition of diaphragms.

While preparing Fig. 11 the writer noted with interest the relationship between the position of maximum moments in the beam and the location of the diaphragms in the structure. When the load is acting over the edge beam, the maximum moment in the beam is found to occur always at midspan of the bridge or in the vicinity of midspan, depending on the position of the diaphragm. Naturally a diaphragm framed at the section nearest the section of maximum moment, which in this case is the load position, transfers the load more effectively. It can be expected, therefore, that under this condition the center diaphragm acts more effectively than diaphragms at the third- or quarter-points. However, when the unit load is acting over beams B or C, the position of maximum moment for the structure with a center diaphragm shifts away from midspan considerably. This position may be as far as the third- or quarter-point of the bridge. Consequently the center diaphragm is some distance from the section of maximum moment, or the load position, and thus can not be more effective than the diaphragms at the third- or quarter-points. In Fig. 11 it can be seen that for beam C and a very stiff diaphragm, the maximum moment for the case with diaphragms at third-points becomes slightly less than that with diaphragm at the midspan.

Except for the slight deviation discussed above, it may be concluded that, for all three beams and three positions of diaphragms shown in Fig. 11, the trend of the curves indicates that the structure with the diaphragm at midspan yields the least moment; the moment becomes larger for diaphragms at the third-points, and largest for diaphragms at the quarter-points.

Similar curves showing the relation between the position of the diaphragm and the moment in the beams under 4-wheel loading are given in Fig. 12. Under truck loading, it is necessary to assume a spacing of the beams in the bridge before the computations for moment can be made. Consideration is given to beam spacings of 5, 6, 7, and 8 ft. For 5 ft. beam spacings, the outer wheel is permitted to come directly over the edge beam in order to produce maximum moment. For beam spacings of 6, 7, and 8 ft., the outer wheel is assumed to be at least 2 ft. from the edge beam. The variations of moment in the beams for different relative diaphragm stiffnesses for these two cases are shown in Figs. 12 (a), (b), (c) and (d). It is to be noted that the value of $\underline{r}$ indicated on each curve represents the relative stiffness of the single center diaphragm or the relative stiffness of each of the diaphragms at the third- or quarter-points. As mentioned before, the computation of moment for 4-wheel loading generally involves some approximation because the influence lines from which the moments are calculated are approximated in the regions between the beams. This approximation may cause some points to deviate from the true curve. In general, the curves were smoothed and cross-checked for consistency in all cases studied. The trend of the curves was found to be consistent.

Fig. 12 shows one interesting fact; whereas, for the intermediate and the center beams, the curves showing the relationship between the moments and the position of diaphragms indicate a similar trend as that for unit loading, the edge beam exhibits an entirely different behavior. Not only does the position of diaphragm at midspan yields the maximum moment in the three positions shown, but the addition of diaphragm increases the moments in the beam under the truck loading. This comparison brings out the major importance of studying the structure under truck loading. This peculiar behavior of the edge beam will be more fully discussed in Section 13.

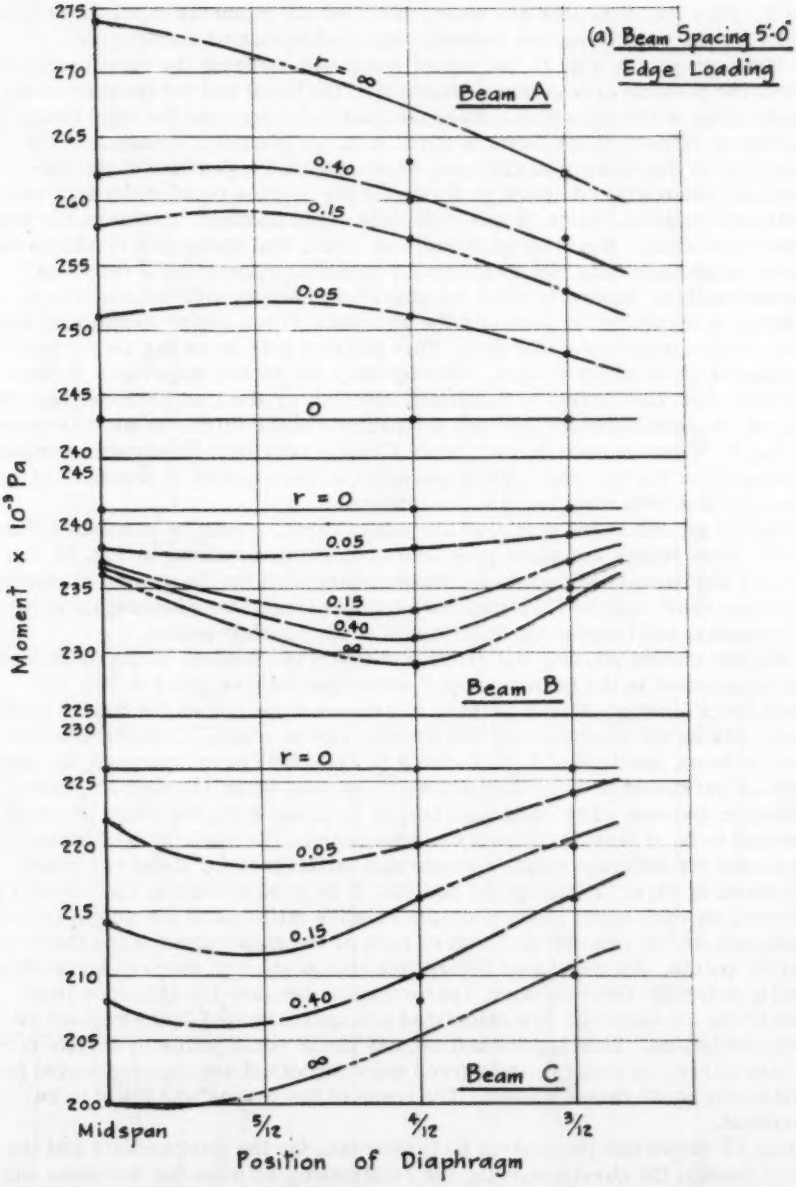


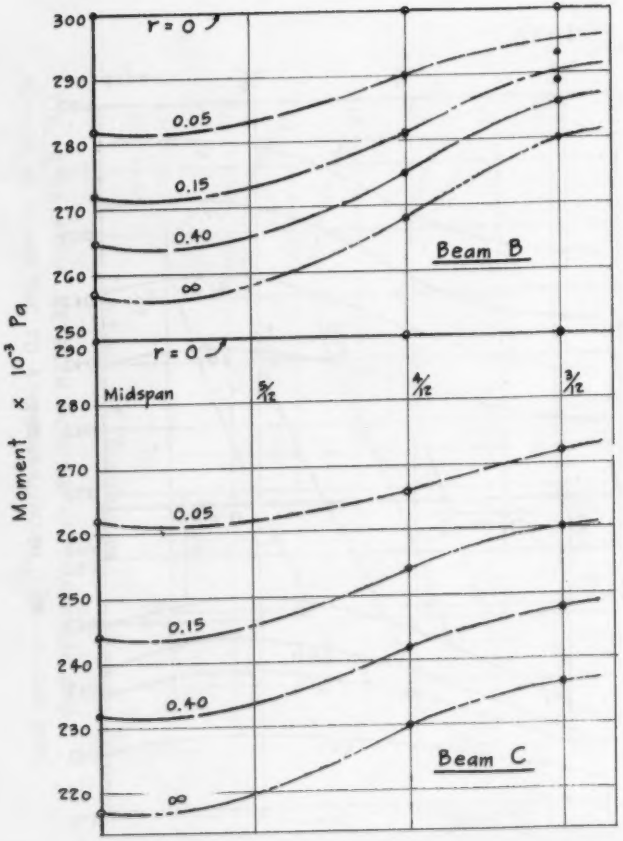
FIG. 12 RELATION BETWEEN POSITION OF DIAPHRAGM AND MOMENT IN BEAMS DUE TO 4-WHEEL LOADING ($^{b}/_{a}$ = 0.1 AND H=10)

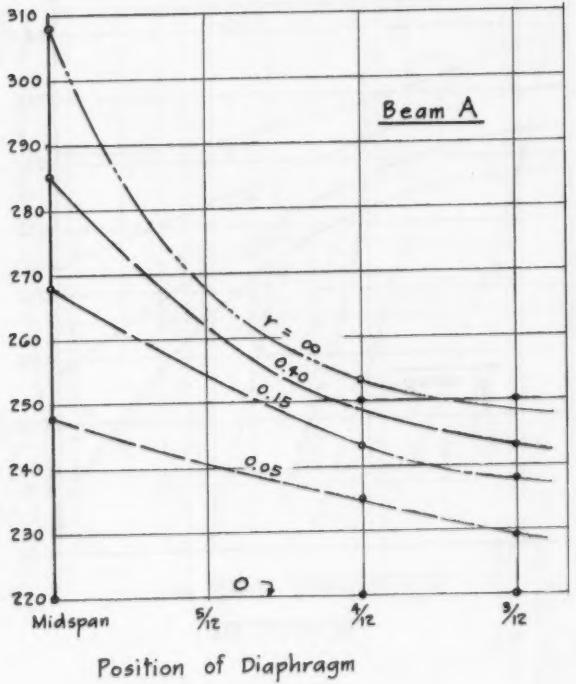
FIG. 12 (CONT'D)

(b) Beam Spacing 6'-0"

FIG. 12 (CONT'D)

(c) Beam Spacing 7'-0"





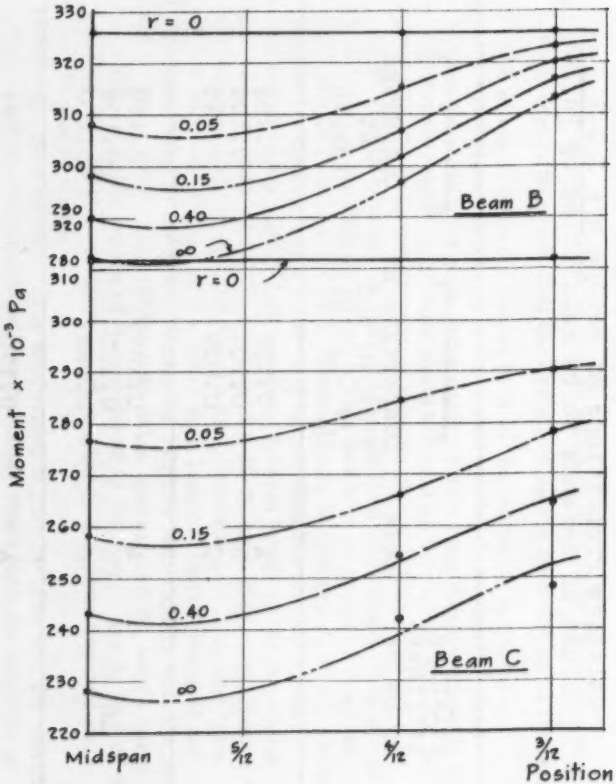
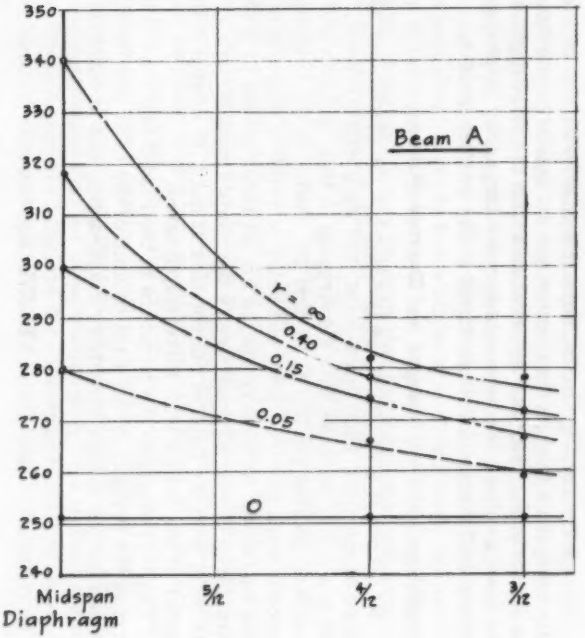


FIG. 12 (CONT'D)

(d) Beam Spacing 8'-0"



Except for the edge beam, the same conclusion for unit loading may generally be applied here in the comparison for the three positions of diaphragms as shown in Fig. 12; that the structure with the diaphragm at midspan yields the least moment, the moments become larger for diaphragms at the third-points, and largest for diaphragms at the quarter-points.

Diaphragms at Both Midspan and Quarter-Points

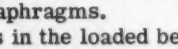
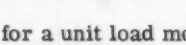
So far the discussion of the positions of the diaphragm has been limited to those of midspan, each third-point, and each quarter-point. In the analyses for b/a = 0.1, the case with diaphragms at both midspan and quarter-points has been treated for H = 5, 10, and 20. Owing to the complexity of the computational work involved, only one value of relative diaphragm stiffness, r = 0.15, was analyzed. Results computed for all three values of H indicated that the structures displayed consistently the same characteristics. It appeared possible then, with the limited information at hand, to interpret the general behavior of the bridge with three diaphragms.

Table I compares the moments in the loaded beam for a unit load moving across the bridge. In the first column of the table are listed the moment values for the bridge with three diaphragms, one at midspan and one each at

TABLE I

COMPARISON OF MAXIMUM MOMENTS IN BEAMS DUE TO UNIT LOAD

Relative Proportions of Bridge b/a = 0.1
Values of moments to be multiplied by Pa

Relative Stiffness of Beams, H	Beam	 Diaphragms at Midspan and 1/4 - points	 Diaphragm at Midspan
5	A	0.159	0.159
	B	0.101	0.101
	C	0.089	0.092
10	A	0.167	0.168
	B	0.105	0.109
	C	0.093	0.100
20	A	0.172	0.174
	B	0.108	0.116
	C	0.096	0.109

quarter points, each with a relative stiffness equal to 0.15. The second column indicates moments for bridge having one center diaphragm, the stiffness of which is equal to that of the central one in the bridge with three diaphragms. It is surprising to note that the maximum controlling moments in beam A check closely for these two cases for all three values of H. This indicates that in a bridge which already has a diaphragm at midspan, the addition of diaphragms at the quarter-points will not help to reduce appreciably the maximum moment. However, it may be seen that the moments in beams B and C are reduced slightly with the addition of extra diaphragms.

Similar comparisons for moments under 4-wheel loading are made in Table II for beam spacings of 6, 7 and 8 ft. Again, the values of moment are surprisingly close for the two cases, as they were under unit load.

It may then be concluded that under both unit load and standard truck loads the effect on the moment of beams in a bridge with three diaphragms, one at midspan and two at the quarter-points, can be approximated by the assumption that the structure has only one diaphragm at midspan with those at quarter-points omitted. In other words, as far as the maximum moment is concerned, the addition of diaphragms at each quarter-point of a bridge which already has a midspan diaphragm will not alter appreciably the behavior of the bridge.

It follows from this conclusion that for construction purposes if diaphragms other than the one at the midspan are required for long span I-beam bridges, they may be made of nominal size and spaced at not more than 25 ft. apart as specified by AASHO.

12. Relative Stiffness of Beams to Slab

The relative stiffness of beams to slab which is defined by the expression $H = \frac{E_b I_b}{aN}$ is one of the important factors affecting the behavior of an I-beam bridge. The effect of different values of H on the moments in beams of a bridge without diaphragms has been discussed earlier in Section 4. It is obvious from the discussion that, for a bridge with a high H corresponding to a structure with a relatively flexible slab, the load on the bridge will not be distributed effectively to the supporting beams. Since a thick roadway slab serves as an effective means of distributing the load that comes on it, the advantage of framing diaphragms in the bridge can best be realized when the slab of the bridge is relatively thin. Consequently in this paper, it was intended to analyze bridges having a value of H larger than the normal amount. Newmark stated in his paper[7] on design of I-beam bridges that in general the value of H for representative designs of a 60-ft. span may range from 3 to 8 for non-composite beams, from 5 to 15 for composite I-beams, and from 15 to 30 for concrete T-beams; for other span lengths, the values of H are roughly proportional to the span, increasing slightly less rapidly than the span increases. For a 60-ft. span bridge with beam spacing of 6 ft., which corresponds to $b/a = 0.1$, the values of H considered in this paper are 5, 10, and 20. These values may be seen to correspond roughly to the three types of construction mentioned above.

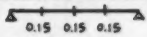
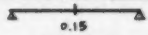
From the formula by which H is expressed, it may be seen that the computation of H is subject to considerable uncertainty since both the modulus of elasticity and moment of inertia of the concrete slab are involved. Neither of these quantities can be determined accurately. The modulus of elasticity of concrete depends on several factors, such as the aging of the concrete, the

TABLE II

COMPARISON OF MAXIMUM MOMENTS IN BEAMS DUE TO 4-WHEEL LOADING

Relative Proportions of Bridge b/a = 0.1
Values of moments to be multiplied by Pa

Beam spacing = 6'- 0"

Relative Stiffness of Beams, H	Beam	 Diaphragm at Midspan and 1/4 - Points	 Diaphragm at Midspan
5	A	0.229	0.223
	B	0.243	0.240
	C	0.226	0.230
10	A	0.225	0.226
	B	0.242	0.240
	C	0.236	0.229
20	A	0.228	0.231
	B	0.245	0.252
	C	0.239	0.242
Beam spacing = 7'- 0"			
5	A	0.264	0.262
	B	0.270	0.271
	C	0.238	0.244
10	A	0.264	0.268
	B	0.271	0.272
	C	0.256	0.244
20	A	0.270	0.276
	B	0.273	0.279
	C	0.257	0.264
Beam spacing = 8'- 0"			
5	A	0.295	0.291
	B	0.292	0.294
	C	0.246	0.257
10	A	0.299	0.300
	B	0.304	0.298
	C	0.269	0.258
20	A	0.304	0.307
	B	0.300	0.298
	C	0.268	0.280

type of aggregate, the plastic flow of concrete, etc. The moment of inertia of the concrete depends on the state of the slab under load. When the slab cracks from moments in the transverse direction, the relative flexural stiffness of slab is reduced at the section where cracking occurs. However, the average transverse stiffness is only slightly reduced by cracking since there are usually fairly large sections of slab that still remain whole. For this reason and to simplify the computation, the value of I used in determining H has been calculated for a plain concrete slab of full depth h, without cracks. The increase in stiffness resulting from the presence of transverse steel may offset to some extent the decrease caused by cracking. This choice of I for the slab in determining H has been justified by the results of tests.

In Fig. 13 and 14 are shown the percentage change of maximum moment under standard truck loads for bridges having $b/a = 0.1$ and various values of H and relative diaphragm stiffnesses. Since the effect on edge beam will be treated separately in Section 13, the maximum moments shown in the figures were obtained for either the intermediate or the central beam in which the maximum occurs, and the change in moment was computed with respect to the same bridge, but without diaphragms. Diaphragms at midspan and at each third-point in bridges with beam spacings ranging from 5 to 8 ft. are considered.

As an illustration to show how much the reduction would be in a 60-ft. span bridge with a beam spacing of 6 ft., corresponding to $b/a = 0.1$, consider the bridge with diaphragms at each third-point, each having a stiffness 10 per cent that of a beam. The maximum moment is reduced approximately 4, 6, and 16 per cent from that in the same bridge without diaphragm, for $H = 5$, 10, and 20, respectively. If for the same bridge, the diaphragm is placed at midspan and is of a stiffness equal to 10 per cent that of a beam, the reduction would be approximately 4, 7, and 13 per cent, for $H = 5$, 10, and 20, respectively. This illustration would indicate that for a bridge with composite construction, the reduction in live load moment in the interior beams could be of an appreciable amount even with the addition of a relatively flexible diaphragm. This advantageous effect of diaphragms may be utilized in the design of these beams.

It may be seen from Figs. 13 and 14 that, where the reduction of moments is pronounced in the cases with the beam spacings from 6 to 8 ft., for beam spacing of 5 ft. and under edge loading the effect of diaphragms in reducing the moments in these beams becomes insignificant even in the bridge with a high value of H.

Another interesting phenomenon to be observed in both Figs. 13 and 14 is that for all three values of H the reduction of moment remains almost constant when the relative diaphragm stiffness (r) exceeds 0.40. This phenomenon has been discussed for the particular value of $H = 10$ in Section 10 and obviously the results shown here would confirm our previous conclusion that, for $b/a = 0.1$ and H in the range of 5 to 20, a diaphragm with an EI of 40 per cent that of each beam is almost as effective as an infinitely stiff one in reducing the maximum moment in the bridge. It can also be seen from Figs. 13 and 14 that the range of r from 0.05 to 0.10 represents a region when a large reduction of moment may be resulted by the introduction of a flexible diaphragm; for higher value of H, corresponding to composite structures, the more flexible diaphragm (say, $r = 5\%$) is relatively more effective. From this consideration and the practical aspects of construction of diaphragms, this range of $r = 0.05$ to 0.10 has been recommended for use in practical design.

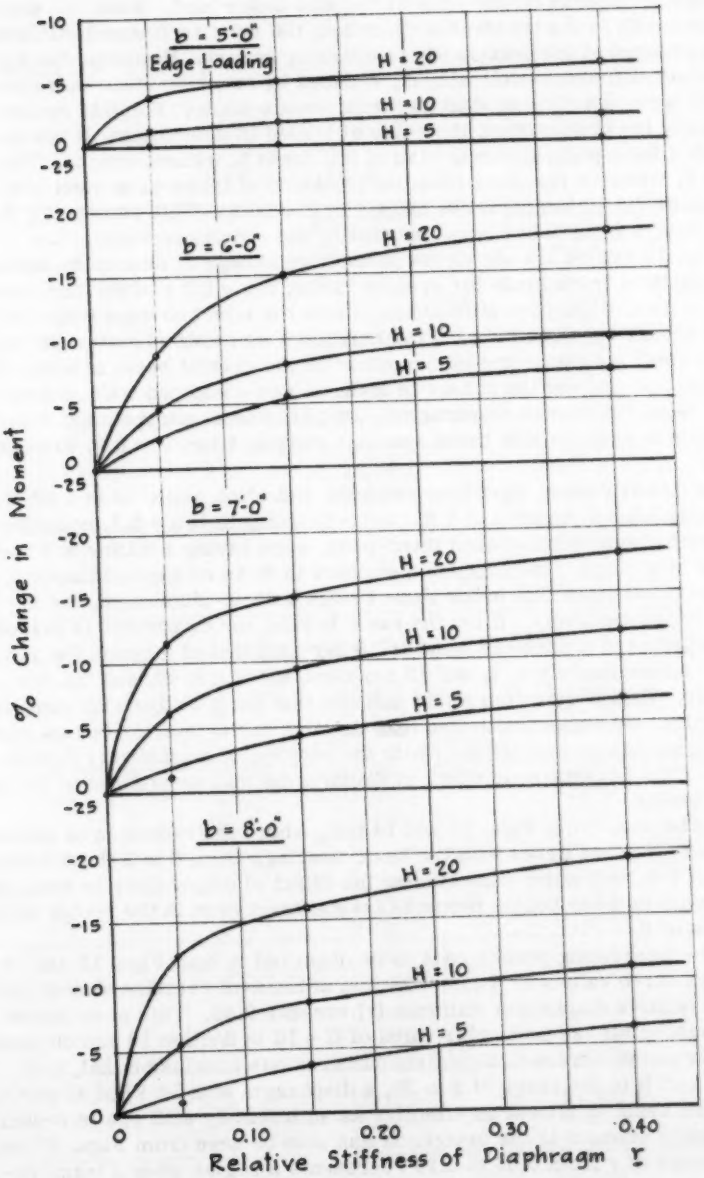


FIG.13

REDUCTION OF MOMENT IN INTERIOR BEAMS UNDER 4-WHEEL LOADING DUE TO ADDITION OF DIAPHRAGM AT MIDSPAN ($b/a = 0.1$)

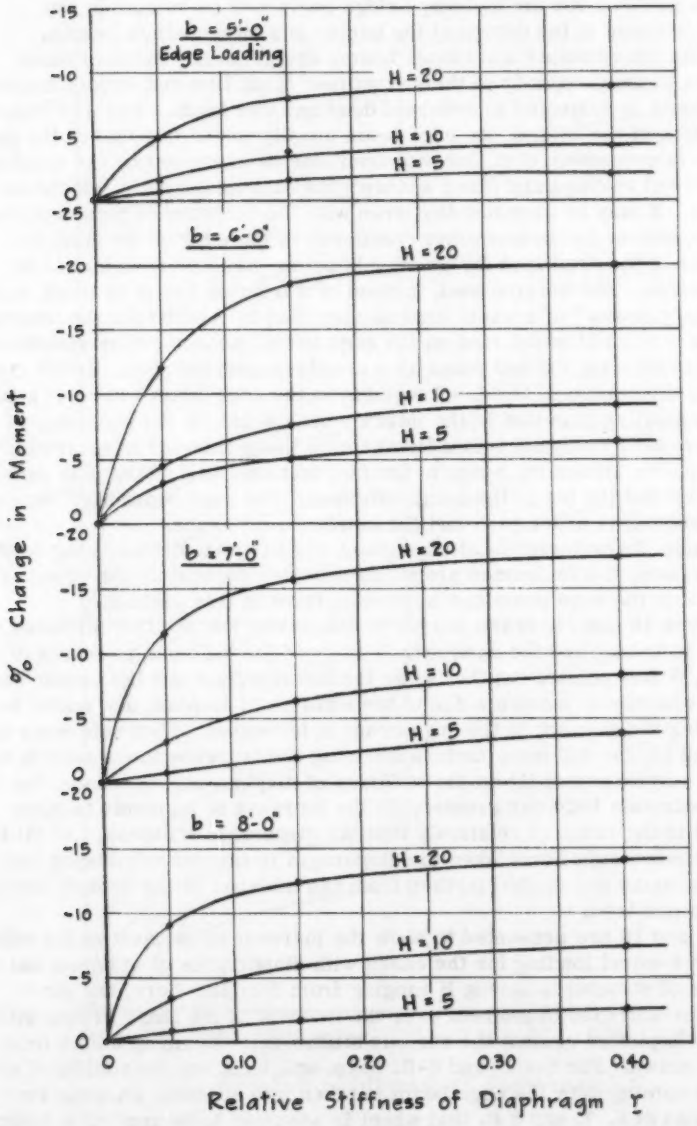


FIG. 14 REDUCTION OF MOMENT IN INTERIOR BEAMS UNDER 4-WHEEL LOADING DUE TO ADDITION OF DIAPHRAGMS AT EACH THIRD POINT ($b/a = 0.1$)

13. Behavior of Edge Beam

The design of the edge beam in an I-beam bridge is almost in all instances a separate problem for the highway bridge engineers as related to the problems involved in the design of the center and intermediate beams. Whereas the intermediate and center beams are generally made of same section due to the proximity of their combined dead, live and impact moments, the edge beam is subjected to different dead and live loads. For an ordinary cross section of the bridge, the edge beam usually takes only half of the panel of the concrete roadway slab, but this dead load is increased by the weights of the curb and railings and other accessories such as conduits, electrical cables, etc. It may be assumed that even with the increase of these additional dead loads, unless for an excessive cantilever of the edge of the slab, the moment due to the dead load for an edge beam is still less than that of the interior beams. For the live load, instead of a fraction (equal to beam spacing divided by a constant) of a wheel load as specified by AASHO for the interior beams, the portion of wheel load on the edge beam is taken as the simple span reaction, considering the end panel as a simply-supported slab. Under these provisions, the combined D + L + I moment in the edge beam would, in general, still be smaller than that in the interior beams and, in the past, many bridges have been designed with a lighter edge beam than the interior ones. For a composite structure, owing to the fact that only half of the slab panel could be used for the tee of the composite beam, the edge beam may require a heavier section in order to resist the combined moments.

The design characteristics of edge beam in an I-beam bridge being such as described above, it was deemed proper to consider separately the effects of diaphragms on the edge beam and to present them in this section.

In Sections 10 and 11, in the course of discussing the relative stiffness of diaphragm to beams and the comparison study of the different positions of the diaphragm, it was pointed out that while the intermediate and the center beams showed a reduction of moments due to the addition of diaphragms, under 4-wheel loading the moment in the edge beam is increased. With reference to Figs. 10 and 12, the following factors affecting the increase in moment in the edge beam may be noted; (1) as the stiffness of diaphragms increases, the increase of moments becomes greater, (2) the increase in moments is more pronounced in the range of relatively flexible diaphragm stiffness, and (3) the increase is more significant when the diaphragm is framed at midspan and, as the diaphragms are spaced further from the midspan of the bridge, the increase becomes less.

Figs. 15 and 16 are presented to show the increase of moment in the edge beam under 4-wheel loading for the cases with diaphragms at midspan and at third-points of structures having $\underline{H}$ ranging from 5 to 20. Here, the percentage of the increase of moment over the moment in the same bridge without diaphragms is plotted against the various stiffnesses. Beam spacings from 5 to 8 ft. are shown. For the 5- and 6-ft. beam spacings, the possibility of an outer wheel coming over the edge beam is taken into account, whereas for beam spacings of 6, 7, and 8 ft. that wheel is assumed to be applied at least 2 ft. from the edge beam. As pointed out above, the increase in moment is much less for the bridge with diaphragms at each third-point (Fig. 16) than the one for midspan (Fig. 15). For both cases, the effect is less significant in small beam spacings of 5 and 6 ft. under edge loading condition. However, the increase shown for beam spacings from 6 to 8 ft. in Fig. 15 is of sizable proportion. For instance, in a 60-ft. span bridge with beam spacing of 6 ft.

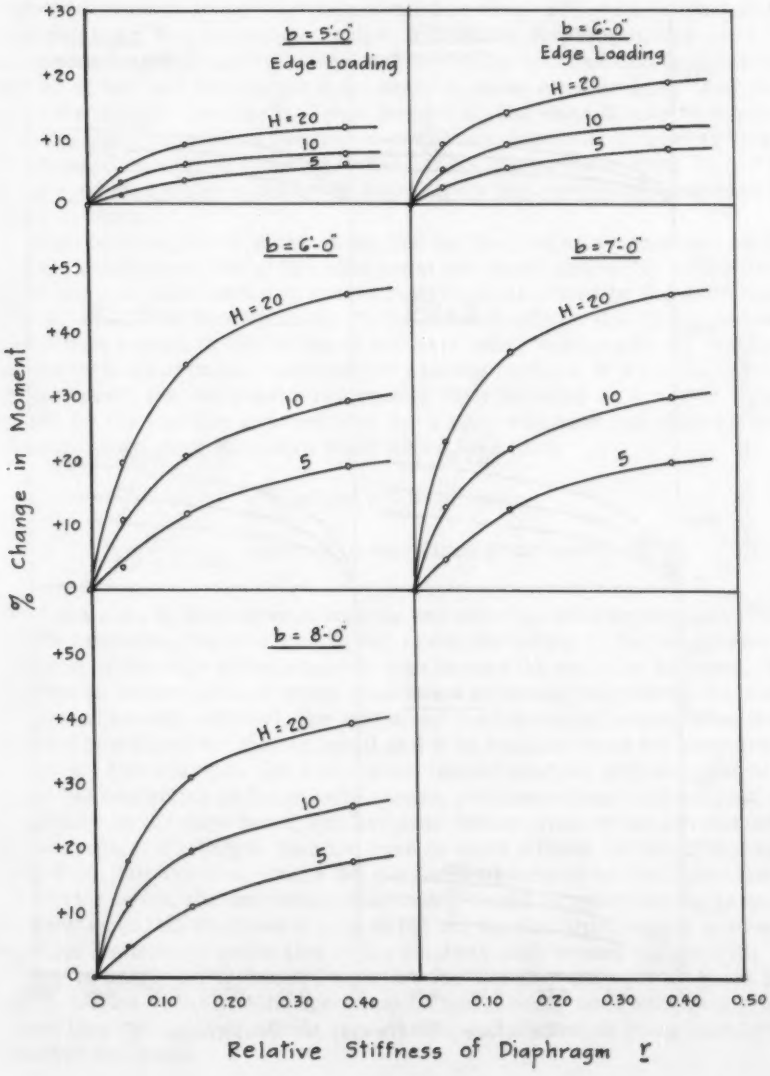


FIG. 15

INCREASE OF MOMENT IN EDGE BEAM UNDER 4-WHEEL LOADING DUE TO ADDITION OF DIAPHRAGM AT MIDSPAN ($b/a = 0.1$)

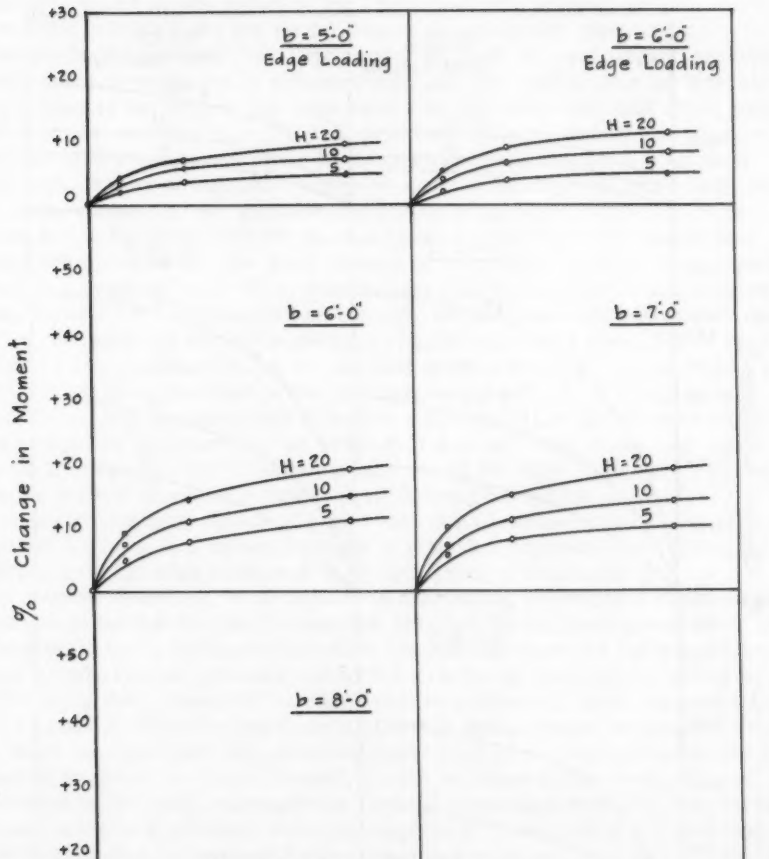


FIG. 16

INCREASE OF MOMENT IN EDGE BEAM UNDER 4-WHEEL LOADING DUE TO ADDITION OF DIAPHRAGMS AT EACH THIRD POINT ($b/a = 0.1$)

and a diaphragm of 10% framed at midspan of bridge, the percentages of increase in moment are 9, 17, and 28, for H = 5, 10, and 20, respectively. For the bridge described above, with two-lane and H = 10, the live plus impact moment would constitute approximately 60% of the total moment. Hence, the increase in moment in the edge beam would be about 10% with respect to the design moment. For composite bridges and stiffer diaphragm, this percentage may be much higher. Overloading of edge beam of this type has been observed in test and statements were made to make edge beam to be of same size as the interior beams.(5) From results of this study it may be concluded that in design of edge beam the live load moment should be increased by a percentage to remedy this harmful effect of the diaphragm. Figs. 15 and 16 may be used as a guide in obtaining quantitively this percentage increase for various bridges.

It must be brought out at this point that all the previous comments on design and recommendation of the edge beam are based primarily on the provisions for load distribution in exterior stringer as stated by the sixth edition (1953) of the ASSHO Specification. In the seventh edition (1957), the following changes with regard to the design of exterior beam were made; (1) For dead load, the weights of curbs, railings and wearing surface, if placed after the slab has cured, may be considered equally distributed to all roadway stringers, and (2) the fraction of wheel load for a span with concrete slab supported by four or more steel stringers shall not be less than:

$$\frac{b}{5.5} \text{ where b = 6' or less;}$$

$$\frac{b}{4.0 + 0.25b} \text{ where b is more than 6' or less than 14',}$$

and b = distance in feet between outside and adjacent interior stringer. Under this new provision, the live load factor is not dependent on the transverse projection of the edge of the roadway slab beyond the exterior stringer. When compared to computation of wheel load based on simply-supported end panel (as done in the sixth edition), the new wheel load factor is larger when the projected portion of the slab is small and it is smaller when the projection is excessive. For example, for a cross-section of roadway with the curb at 2-ft outside the centerline of the exterior beam, corresponding to wheel load acting directly on the edge beam, the live load factor given by the seventh edition is an average of 8% larger than the ones by sixth edition for beam spacings of 6 to 8 ft. For the case where the edge of the curbe is on the centerline of the exterior beam, the new wheel load factor would be considerably larger. The purpose of this discussion is to bring out the fact that, except in cases where the transverse projection of the roadway slab beyond the exterior stringer is large, the live load factor for the exterior stringer as given by the seventh edition of the AASHO Specification has already been arbitrarily made greater than that derived by the conventional calculation of using simply-supported end panel.

CONCLUSION

Throughout the presentation of the paper, emphases have been given to I-beam bridges with a relative proportion of b/a = 0.1 and H in the range of 5 to 20. As pointed out in Section 6, these proportions were chosen to represent

a practical range of longer span I-beam bridges covering both the composite and non-composite construction. The following conclusions may be drawn from the results of this analytical study:

(1) A diaphragm with a relative stiffness of 40 per cent that of each beam is almost as effective as an infinitely stiff one in reducing the maximum moment in the bridge. For practical considerations, a flexible diaphragm with a relative stiffness in the range of 5 to 10 per cent of that of the beam is recommended for use in design; the lower limit (5%) being favored for bridges with a high H, corresponding to composite construction and the higher limit (10%), for non composite structures.

(2) Under both unit and standard truck loads, the addition of diaphragm reduces the maximum moments in the interior beams. In general, the moment in these beams in the structure with the diaphragm at midspan yields the least value; the moment becomes larger for diaphragms at the third-points, and largest for diaphragms at the quarter-points.

(3) Although under unit load the moment in the edge beam is reduced, under 4-wheel truck loads the addition of diaphragm causes an increase in the moment. The increase is more significant for the case with diaphragm framed at midspan and becomes less as they are spaced further from the midspan. Percentages of increase in moments in edge beam for various bridges have been shown in Figs. 15 and 16. Live load moment in edge beam, computed on the assumption of simply-supported end panel, should be increased in design by a percentage similar to those shown in Figs. 15 and 16 to provide for this harmful effect of the diaphragms.

(4) The effect of diaphragms on the moment of beams in a bridge with three diaphragms, one at midspan and one each at the quarter-points can be approximated by the assumption that the structure has only one diaphragm at midspan with those at the quarter-points omitted. In other words, as far as the maximum moment is concerned, the addition of diaphragms at each quarter point of a bridge which already has a midspan diaphragm will not alter appreciably the behavior of the bridge. From this conclusion it follows that, for construction purposes, if diaphragms other than the one at midspan are required for long spans, they may be made of nominal size and spaced at not more than 25 ft. apart as specified by AASHO.

ACKNOWLEDGMENT

The major part of the study presented in this paper is based on a report[4] which formed a part of an investigation of the reinforced concrete slab as conducted by the Engineering Experiment Station of the University of Illinois in cooperation with the Division of Highways of the State of Illinois and the Bureau of Public Roads of the United States Department of Commerce. The writer is sincerely thankful to Professors N. M. Newmark and C. P. Siess, both M. ASCE, for their guidance, encouragement, and many valuable suggestions during the performance of the work.

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Journal of the
STRUCTURAL DIVISION
Proceedings of the American Society of Civil Engineers

ANALYSIS OF TWO-COLUMN SYMMETRICAL BENTS

B. R. Cooke,[1] J. M. ASCE

SYNOPSIS

A method utilizing cantilever moment distribution, symmetry, and antisymmetry for the direct solution of many symmetrical two-column structures with loads between joints is presented. A symmetrical multi-story bent with loading on the columns is analyzed by equating the loading to cases of symmetry and antisymmetry.

SYMBOLS

V_{AB} = shear carried by member AB at joint A

M_{AB} = moment acting on joint A from member AB

M_{fAB} = fixed-end moment acting on joint A from member AB

M_{cAB} = cantilever fixed-end moment acting on joint A from member AB

SIGN COVENTION

Positive moments will be defined as those which tend to rotate the joint clockwise.

INTRODUCTION

The direct solution of a multi-story two-column symmetrical bent with any horizontal loading on the columns or at the girders is possible by cantilever

Note: Discussion open until October 1, 1959. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. Paper 2037 is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 85, No. ST 5, May, 1959.

1. Resident Engr., Texas Highway Dept., Monahans, Tex.

moment distribution[2] with cantilever fixed-end moments and ordinary moment distribution. This is accomplished by equating the loading to cases of symmetry and antisymmetry and summing the results of the separate moment distribution solutions.

Cantilever Moment Distribution

A direct moment distribution method for the solution of joint translation problems applicable to many structures with identical chords or columns is given in a paper[2] by L. E. Grinter and C. H. Tsao. This cantilever moment distribution depends upon a known shear acting on each column. The structure is allowed to translate and as the joints are released, the carry-over factor is made such that there is no change in column shear.

In Fig. 1, a column is allowed to translate due to the applied moment, M. The slope, θ, caused by this moment is ML/EI as compared to ML/4EI had the column not been allowed to translate. Hence, for cantilever moment distribution the effective stiffness factor of the column is only 1/4 that of a member that is not allowed to translate. The carryover is -1 as compared to +0.5 for moment distribution without joint translation.

Cantilever Distribution For A Simple Bent

A solution of a simple bent using cantilever moment distribution is given in Fig. 2. The fixed-end moment for translation without rotation is 100 ft. kips at each end of each column (i.e., a total of 20 kips times 20 ft. = 400 ft. kips). The top member has an effective stiffness of 6EK due to the antisymmetrical moments caused by sidesway. The column has an effective stiffness of 1/4(4EK) or EK. The distribution factors at joint A are 1/7 for the column and 6/7 for the top member. The unbalanced moment of +100 causes distributions of -1/7(100) = -14.3 to the column and -6/7(100) = -85.7 to the top member at joint A. The carryover to B is -1(-14.3) or +14.3. The total moments shown are the final moments.

Cantilever Fixed-End Moments

Fig. 3 is a simple symmetrical bent with antisymmetrical loads on the columns. From this antisymmetry the shear at the base of the columns is known. No horizontal load is transferred by the top girder. If a fixed-end moment is used that allows the frame to translate without horizontal load being transferred by the girder, cantilever moment distribution may be used.

Fig. 4 is a beam with one end fixed and the other end guided so as to retain a slope of zero while not developing any end reaction at B. If the guided end is allowed to rotate and not develop an end reaction, then the carryover from B to the fixed-end A is -1. Thus if cantilever fixed-end moments (one end fixed and the other end guided) are used, a symmetrical bent having antisymmetrical loading may be analyzed by cantilever moment distribution. Fig. 5 gives equations for cantilever fixed-end moments for the cases shown.

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2. "Joint Translation by Cantilever Moment Distribution", by L. E. Grinter and C. H. Tsao, Proceedings, ASCE, Vol. 79, Separate No. 298.

FIGURE 1

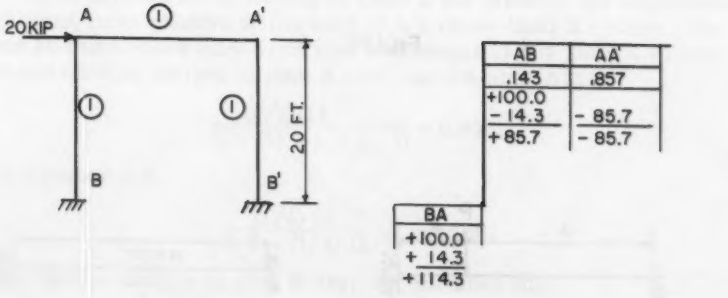


FIGURE 2

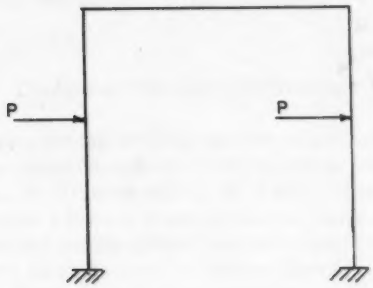


FIGURE 3

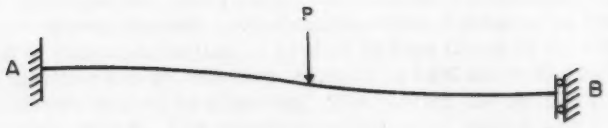
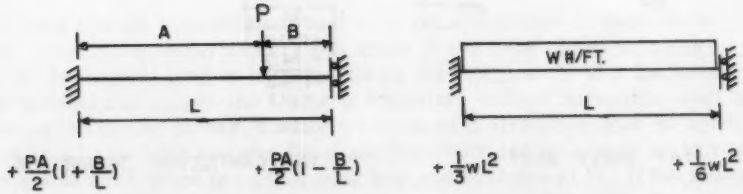


FIGURE 4



CANTILEVER FIXED-END MOMENTS

FIGURE 5

Multi-Story Bent Solution by Symmetrical and Antisymmetrical Loading

Fig. 6(a) is a two-story bent with loading as shown. The circled numbers are relative values of the stiffness of the members. The loading may be equated to a symmetrical (Case I) and antisymmetrical (Case II) loading (Fig. 6(b) and (c)). By solving these two equivalent loadings the results may be added to give a solution for the original loading on the frame.

In Case I there is no joint translation due to the symmetry of the loading. The girders due to the symmetrical end moments have a stiffness of 2EK or a relative stiffness of 1/2 their K values or 1/2(1) = 1/2.

The distribution factors at joint A are for member AA:

$$\frac{1/2}{2/3 + 1/2} = 0.429,$$

and for member AB:

$$\frac{2/3}{2/3 + 1/2} = 0.571.$$

The distribution factors at joint B are; for member BB:

$$\frac{1/2}{2/3 + 2/3 + 1/2} = 0.272,$$

and for members BA and BC:

$$\frac{2/3}{2/3 + 2/3 + 1/2} = 0.364.$$

Usual fixed-end moments are used. The solution of Case I is given in Fig. 7 (a).

For the antisymmetrical loading of Case II the girders, for antisymmetrical moments, have relative stiffnesses of 3/2 times their K values. The columns as cantilevers have corrected stiffnesses of 1/4 their K values.

The distribution factors at joint A are: for member AA;

$$\frac{(3/2)(1)}{3/2\ (1) + 1/4\ (2/3)} = 0.900,$$

and for member AB;

$$\frac{(1/4)\ (2/3)}{3/2 + (1/4)\ (2/3)} = 0.100.$$

The distribution factors at joint B are: for member BB;

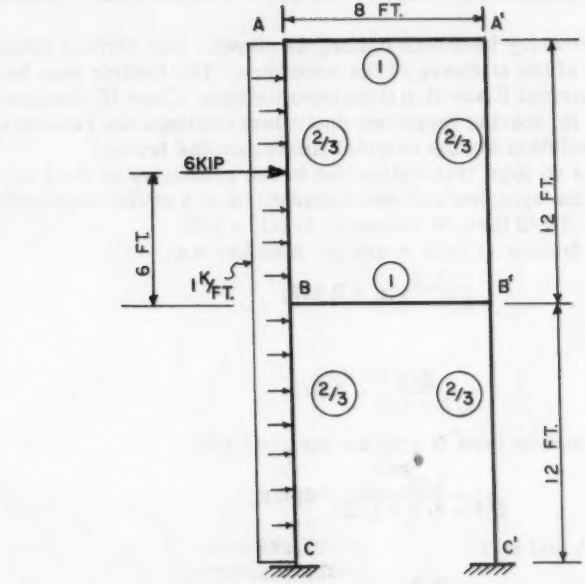
$$\frac{(3/2)\ (1)}{(3/2)\ (1) + (1/2)\ (2/3) + (1/2)(2/3)} = 0.818,$$

and for members BA and BC;

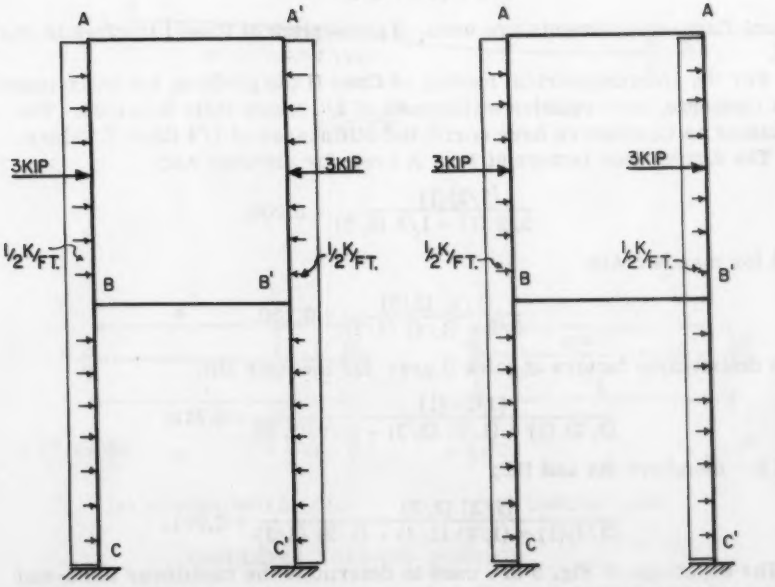
$$\frac{(1/2)\ (2/3)}{(3/2)(1) = (1/2)\ (2/3) + (1/2)\ (2/3)} = 0.091.$$

The equations of Fig. 5 are used to determine the cantilever fixed-end moments. The cantilever fixed-end moment at AB due to the concentrated load is as follows:

$$\frac{Pa}{2}\left(1 - \frac{b}{L}\right) = \frac{(3)6}{2}\left(1 - \frac{6}{12}\right) = +4.5 \text{ ft. kips}.$$



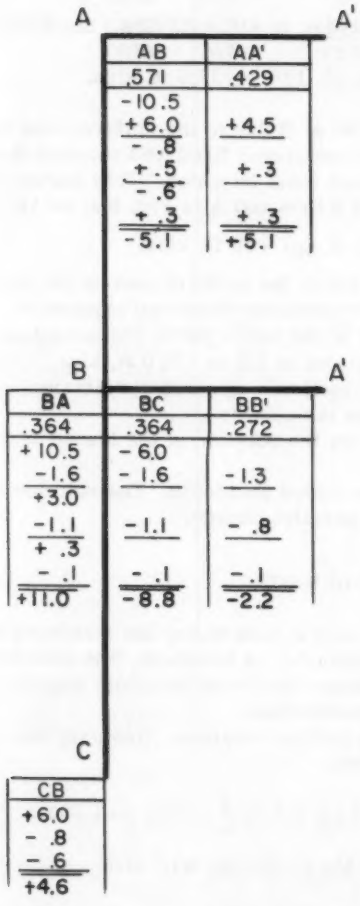
(A) LOADED BENT

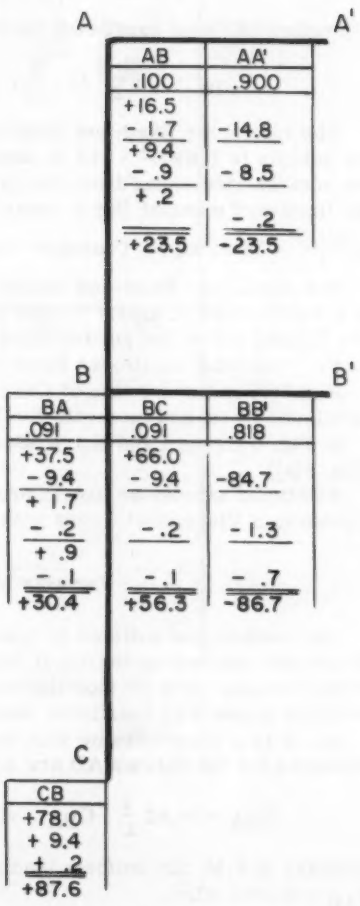


(B) SYMMETRICAL LOADING CASE I

(C) ANTISYMMETRICAL LOADING CASE II

FIGURE 6





(A) SOLUTION OF CASE I (B) SOLUTION OF CASE II

FIGURE 7

The cantilever fixed-end moment at AB due to the uniform load is as follows:

$$1/6wL^2 = 1/6\ 1/2\ (12^2) = +12.0 \text{ ft. kips.}$$

Therefore the total cantilever moment, M_{cAB}, is +16.5 ft. kips. Similarly

$$M_{cBA} = + \frac{(3)\ 6}{2} (1+ \frac{6}{12}) + 1/3(1/2)\ (12^2) = +37.5 \text{ ft. kips.}$$

The cantilever fixed-end moment for BC at B due to the uniform load on the column is $1/6wL^2 = +12$ ft. kips. The cantilever fixed-end moment due to the accumulated shear from the panel above must be added. This portion of the fixed-end moment (for a shear load of 9 kips and A in Fig. 5(a) as 12 ft.) is $\frac{(9)12}{2} = 54$ ft. kips. Therefore the total $M_{cBC} = 66$ ft. kips.

The cantilever fixed-end moment CB due to the uniform load on the column is $1/3wL^2 = +24$ ft. kips. To this must be added the fixed-end moment of +54 ft. kips due to the accumulated shear in the above panel, calculated as above. The total cantilever fixed-end moment at CB is +78.0 ft. kips.

Fig. 7(b) is the solution of Case II by cantilever moment distribution using cantilever fixed-end moments for loads on the columns.

In Fig. 8 Cases I and II are added to give the solution of the loaded frame (Fig. 6(a)).

Additional stories or panels present no added problems. The solution also applies to a Vierendeel Truss with equal parallel chords.

Variable Moment of Inertia

The method just outlined is applicable to the case where the members are of variable moment of inertia if frame symmetry is retained. The stiffness of the columns must be modified and ordinary fixed-end moments may be modified to use with cantilever moment distribution.

Fig. 9 is a simple frame with variable section columns. Ordinary beam constants for the column AB are as follows:

$$K_{BA} = 20.61 \frac{I}{L}\ ;\ C_{BA} = +0.316;\ K_{AB} = 5.42 \frac{I}{L}\ ;\ C_{AB} = +1.200.$$

Ordinary F.E.M. for uniform loading are $M_{BA} = 0.1451\ wL^2$ and $M_{AB} = 0.0561\ wL^2$.

Fig. 10 is the resulting M/EI diagram due to a unit rotation at B with translation. Equating the area of the $\frac{M}{EI}$ diagram to unity results in a moment of $1.778\ \frac{EI}{L}$ which is the stiffness of AB. Fig. 11 is the moment diagram resulting from a unit rotation at each end, which is equivalent to translating the column without rotation. The shear at B resulting from a uniform load with end A and B fixed (no translation) is 0.411 wL. If the column is translated without end rotation by an equal shear acting in an opposite direction to the 0.411 wL at B, and the resulting end moments are added to the ordinary fixed-end moments, cantilever fixed-end moments are obtained. Fig. 11, which is equivalent to translating the column without end rotation, gives the moment at A in terms of M_B. Therefore by a moment equation at B:
$1.440M_B = 0.411\ wL^2$; $M_B = 0.2854wL^2$ and $M_A = (0.440)\ .2854wL^2 =$

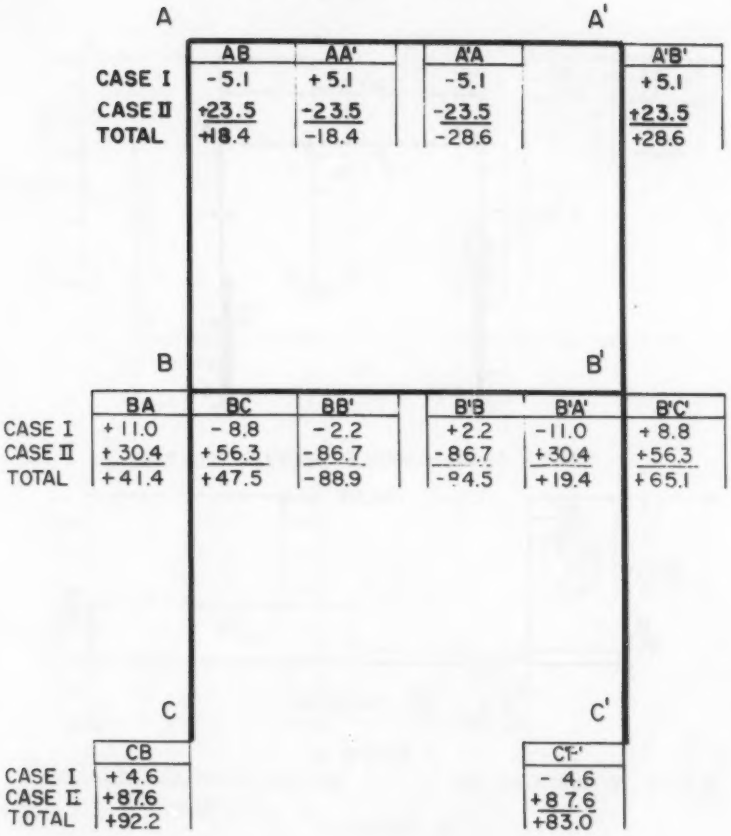


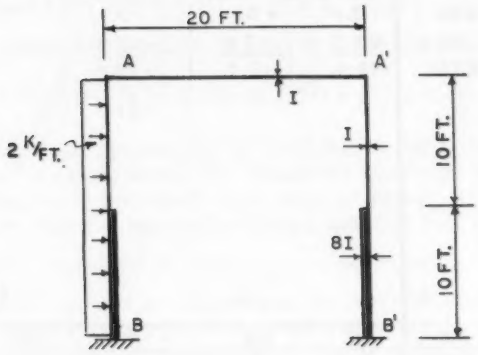
FIGURE 8

$0.1256wL^2$. Adding these moments with regard to signs to the ordinary fixed-end moments results in the following cantilever fixed-end moments: $M_B = +0.4305wL^2$; $M_A = +0.0695\ wL^2$.

Fig. 12(a) is the symmetrical loading. Ordinary stiffness, fixed-end moments, and carryovers are used in the solution (Fig. 12(b)).

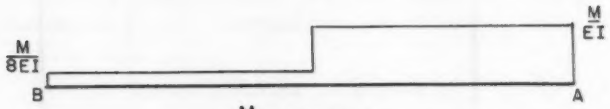
Fig. 13(a) is the antisymmetrical loading. The modified stiffness of $1.778\ \frac{I}{L}$ of the columns, the cantilever fixed-end moments, and the -1 carry-over are used for the solution in Fig. 13(b).

In Fig. 14 the symmetrical and antisymmetrical loading solutions are added to give the final solution.



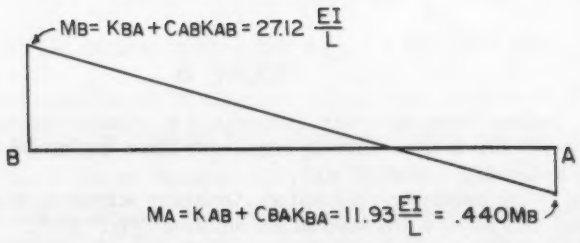
SIMPLE BENT—VARIABLE SECTION COLUMNS

FIGURE 9



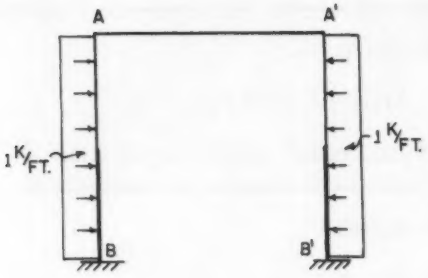
$\frac{M}{EI}$ DIAGRAM

FIGURE 10



UNIT ROTATION AT EACH END

FIGURE 11



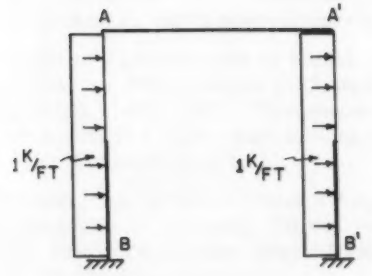
(A) SYMMETRICAL LOADING CASE I

AB	AA'
.730	.270
-22.44	
+16.38	+6.06
- 6.06	+6.06

BA
+58.04
+19.66
+77.70

(B) SOLUTION OF CASE I

FIGURE 12



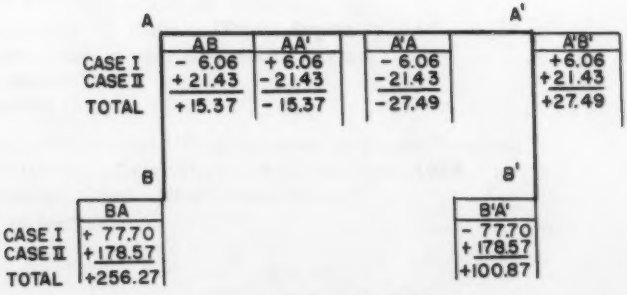
(A) ANTISYMMETRICAL LOADING CASE II

AB	AA'
.229	.771
+ 27.80	
- 6.37	- 21.43
+ 21.43	- 21.43

BA
+172.20
+ 6.37
+ 178.57

(B) SOLUTION OF CASE II

FIGURE 13



CASE I + CASE II

FIGURE 14

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MOMENTS IN CONTINUOUS BEAMS ON FLEXIBLE SUPPORTS[a]

Closure by Robert A. Williamson

ROBERT A. WILLIAMSON,[1] A. M. ASCE.—Mr. Bergman's excellent discussion is sincerely appreciated. His presentation permits a comparison between the successive approximation technique of the paper and an approximate method based on finite differences. The latter solution provides acceptable values of deflections, and, with the use of a sufficient number of additional pivotal (nodal) points, will yield essentially correct moments.

The finite difference approach is a powerful method capable of solving a variety of complex problems which are beyond the capacity of less sophisticated methods, through the substitution of simultaneous equations for differential equations. When applied to the example solved in this paper, Mr. Bergman's adaptation of the finite difference method determines deflections in an efficient way by the formulation of a set of such equations which can be solved readily using an iterative technique.

However, the objective is to provide the designer with reasonably accurate moments, and this Mr. Bergman's solution fails to do. The moments are grossly in error, as shown by the following tabulation, which indicates the discrepancies between the moments obtained in the paper by balancing angle changes (Method A), and those derived by statistical considerations from Mr. Bergman's results (Method B). These differences are expressed as percentages of the results of Method A. Also shown is a similar comparison of deflections.

Discrepancies in Moment and Deflection in Percent

Point		A	B	C	D	E
Moments	$100\left(\frac{B-A}{A}\right)$	-48	-43	-61	174	0
Deflections	$100\left(\frac{B-A}{A}\right)$	0	-4.3	-0.3	0.5	2.7

The very large differences in moment, which, for the most part, appear to be caused by seemingly minor discrepancies in deflection of less than 5%, illustrate the extreme sensitivity of bending moments to very slight errors in computed deflections in a problem of this type.

Calculation of accurate bending moments by statics, from the deflections of Method B, apparently requires that these deflections be computed using more

a. Proc. Paper 1631, May, 1958, by Robert A. Williamson.
1. Structural Engr., Holmes & Narver, Engrs.-Constructors, Los Angeles, Calif.

than four pivotal points, because of the approximation which expresses $\frac{d^2y}{dx^2}$ in terms of deflections. This may increase the number of unknowns to the point where there would be little gain, if any, in computational efficiency, over that afforded by the repetitive tabular "bookkeeping" procedure of balancing angle changes. Furthermore, in view of the large errors in moments obtained with four pivotal points, one might question the accuracy of results obtained with even eight pivotal points, which, incidentally, would require solving eight simultaneous equations. It should be noted that the technique of balancing angle changes involves no questions of accuracy except those concerned with the degree of convergence; increased accuracy is a matter of more cycles, not more unknowns.

REFERENCES

1. Numerical Methods in Engineering by M. G. Salvadori and M. L. Baron, Prentice Hall Inc., New York 1952, pg. 147-148.

ANALYSIS OF FINITE BEAMS ON ELASTIC FOUNDATION[a]

Corrections to Discussion by Alexander Dodge
Closure by D. C. Gazis

CORRECTIONS TO DISCUSSION BY ALEXANDER DODGE,[1] M. ASCE.—In the January, 1959, Journal of the Structural Division, on page 168, in the line above Table 2, the correct expression for y is -

$$y = \Delta p \frac{Pb^3}{EI} = 177\ \Delta p,$$

and the reference to correction in this expression, as given on page 189 in the March 1959 Journal of the Structural Division, is hereby deleted.

D. C. GAZIS,[2] J. M. ASCE.—The author wishes to thank the discussers for their very interesting contributions. The alternative method of analysis of finite beams on elastic foundation which has been suggested by Mr. Reddy is very elegant, while the ones suggested by Messrs. Retti and Dodge are excellent examples of efficient and knowledgeable engineering design.

In his own paper the author attempted to present an iteration technique which employs simple concepts and can be duplicated after a minimum of training. The method is quite flexible and effective in handling problems much more complex than the given example. In answer to Mr. Retti's related remark it may be pointed out that any complex loading requires exactly the same amount of labor after the original computation of the "end forces" and moment and shear diagrams of the infinite beam. Moreover, it is sufficient to have the influence lines of those "end forces" in order to reduce any problem to a very simple accounting task. The method is also very easily adaptable to the use of a modern electronic computer. The iteration always converges except for the extreme case of an infinitely rigid beam. This brings up Mr. Dodge's objection to the "arbitrary" distinction between long, medium, and short beams. The answer to this criticism is that this distinction is purely operational and no claim is being made on its being either logical or well defined. In principle it is about as arbitrary as the age-old design assumption of many footings as rigid, in computing foundation reactions. The classification of beams given in the last section of the paper is intended merely as a guide to a designer and it appears to guarantee an accuracy of at least 3%, as may be ascertained by observing the variation of the "carry-over" factors with $\beta\ell$.

Finally, it is unfortunate that in the correction published in No. 1922, there was a typographical error resulting in a reference to a "Movement" diagram.

---

a. Proc. Paper 1722, July, 1958, by D. C. Gazis.

1. Tech. Asst., Corp. of Engrs., Portland, Ore.
2. Sr. Research Scientist, Research Staff, General Motors Corp., Detroit, Mich.

It was not the intention of the author to introduce a new type of a diagram, or to coin a new term for the "Moment" diagram.

GENERALIZATION OF THE SUCCESSIVE APPROXIMATION METHOD[a]

Discussion by Perng-Fei Gou

PERNG-FEI GOU,[1]—Mr. Moliotis has derived a general formula to express the new index of stiffness and formulas expressing the carry-over factor for such frames whose joints of the frames are a small undeformed area. Those formulas can also be derived by the well known slope-deflection equation. The derivations are given below:

Derivation of the Index of Stiffness

In Fig. 1 the bending moments are given by the slope-deflection equations

$$M_a = 2EK\left(2\theta_a + \theta_b + 3\frac{d_a\theta_a}{l}\right) \tag{1}$$

$$M_b = 2EK\left(2\theta_b + \theta_a + 3\frac{d_a\theta_a}{l}\right) \tag{2}$$

Member AB is in equilibrium, then

$$Q_a = Q_b = \frac{1}{l}(M_a + M_b) = 2EK\left[\frac{\theta_a}{l}\left(3+6\frac{d_a}{l}\right)+3\cdot\frac{\theta_b}{l}\right] \tag{3}$$

Member AA' is in equilibrium too, then

$$M_a' = M_a + Q_a d_a \tag{4}$$

Substitute Eq. (1) and (3) into (4), then we obtain

$$M_a' = 2EK\left[2+6\frac{d_a}{l}+6\left(\frac{d_a}{l}\right)^2\right]\theta_a + 2EK\left[1+3\cdot\frac{d_a}{l}\right]\theta_b \tag{5}$$

By the definition of stiffness (that is a moment which, when applied at the pin end of a beam will cause there unit rotation when the far end is fixed against rotation.) then we let θ_b equal to zero and θ_a equal to 1. Therefore,

$$S_A = 4EK\left[1+3\cdot\frac{d_a}{l}+3\left(\frac{d_a}{l}\right)^2\right] \tag{6}$$

If $d_a = 0$, then S = 4EK. That is the general expression of stiffness under the assumption that the joints of the ridge frame are point connections.

Eq. (6) is identical with Eq. (13) which is derived by Mr. Moliotis.

Special Case

(a) $M_b = 0$. That is the case which the joint B is a pin. Then,

$$\theta_b = -\frac{1}{2}\left(1+3\frac{d_a}{l}\right)\theta_a \tag{7}$$

---

a. Proc. Paper 1816, October, 1958, by Panagiotis D. Moliotis.

1. Asst. Civ. Eng. Dept. Cheng Kung Univ., Tainan, Taiwan Free China.

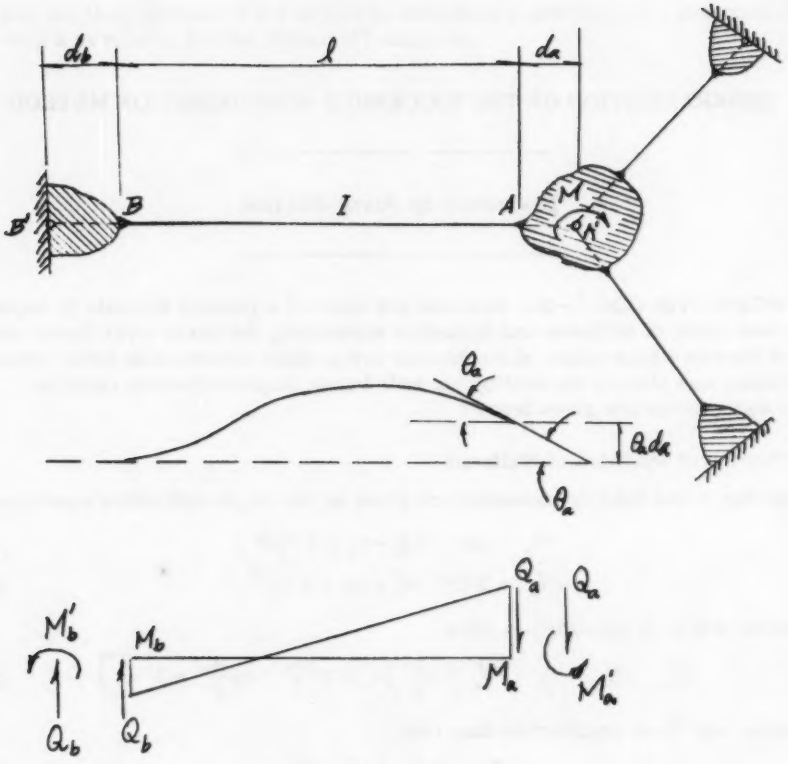

Fig 1.

Substitute (7) in (5) and let $\theta_a = 1$ then

$$\begin{aligned} S &= 2EK\left[2+6\frac{d_a}{l}+6\left(\frac{d_a}{l}\right)^2-\frac{1}{2}-3\frac{d_a}{l}-\frac{9}{2}\left(\frac{d_a}{l}\right)^2\right] \\ &= 2EK\left[1.5+3\frac{d_a}{l}+1.5\left(\frac{d_a}{l}\right)^2\right] \\ &= 3EK\left(1+2\frac{d_a}{l}+\frac{d_a^2}{l^2}\right) \end{aligned} \tag{8}$$

(b) $M_a = 0$. Then

$$\theta_a = -\frac{\theta_b}{-2+\frac{3d_a}{l}} \tag{9}$$

Substitute (9) in (5) and let $\theta_a = 1$. Then we have

$$S = 2EK\left[2+6\frac{d_a}{l}+6\left(\frac{d_a}{l}\right)^2\right] - 2EK\left[2+9\frac{d_a}{l}+9\left(\frac{d_a}{l}\right)^2\right]$$

$$= 2EK\left[-3\left(\frac{d_a}{l}\right)-3\left(\frac{d_a}{l}\right)^2\right] \qquad (10)$$

$$= -6EK\left[\left(\frac{d_a}{l}\right)+\left(\frac{d_a}{l}\right)^2\right]$$

(c) $\theta_a = \theta_b$. That is, the member rotates through the same angle at the far end as the near end and in the same direction. In (5) let $\theta_a = \theta_b = 1$, then

$$S = 2EK\left[2+6\frac{d_a}{l}+6\left(\frac{d_a}{l}\right)^2\right] + 2EK\left[1+3\frac{d_a}{l}\right]$$

$$= 6EK\left[1+3\left(\frac{d_a}{l}\right)+2\left(\frac{d_a}{l}\right)^2\right] \qquad (11)$$

If $d_a = 0$, then $S = 6EK$. We know that is the modified stiffness of those rigid frames whose joints are point connection.

(d) $\theta_a = -\theta_b$. That is the case which the member of a frame rotates through the same angle at the far end as at the near end but in the opposite direction. Or in other words, it is the case of symmetrical condition. In Eq. (5), let $\theta_a = -\theta_b = 1$, then

$$S = 2EK\left[2+6\left(\frac{d_a}{l}\right)+6\left(\frac{d_a}{l}\right)^2\right] - 2EK\left[1+3\left(\frac{d_a}{l}\right)\right]$$

$$= 2EK\left[1+3\left(\frac{d_a}{l}\right)+6\left(\frac{d_a}{l}\right)^2\right] \qquad (12)$$

Eq. (12) is the modified stiffness of a symmetrical condition of point joint rigid frame, if $d_a = 0$.

Derivation of Carry-Over Factors

$$M_b' = M_{ba} + Q_b d_b$$

$$= 2EK\left(2\theta_b+\theta_a+3\frac{d_a\beta_a}{l}\right) + 2EK\left(3\theta_a+6\frac{d_a\beta_a}{l}+3\theta_b\right)\frac{d_b}{l} \qquad (13)$$

$$= 2EK\left[\left(1+3\frac{d_b}{l}\right)\theta_a+\left(2+3\frac{d_b}{l}\right)\theta_b+3\left(1+2\frac{d_b}{l}\right)\frac{d_a\beta_a}{l}\right]$$

By the definition of carry-over factor we obtain,

$$M_b' = M_{ab} M_a' \qquad (14)$$

Substitute (13) and (5) for M and M respectively in (14), then we have

(8)

$$M_{ab} = \frac{\left[\left(1+3\frac{d_b}{l}\right)\theta_a+\left(2+3\frac{d_b}{l}\right)\theta_b+3\left(1+2\frac{d_b}{l}\right)\frac{d_a\beta_a}{l}\right]}{2\left[1+3\frac{d_a}{l}+3\left(\frac{d_a}{l}\right)^2\right]\theta_a+\left[1+3\frac{d_a}{l}\right]\theta_b} \qquad (15)$$

Eq. (15) is the general expression of carry-over factor and it is practically equal to the Eq. (19) which Mr. Moliotis gave.

(9)

Special Cases

(a) $\theta_b = 0$. This is the case which section B is fixed end.

$$M_{ab} = \frac{1}{2}\,\frac{1+3\frac{d_a+d_b}{l}+6\frac{d_a\cdot d_b}{l^2}}{1+3\frac{d_a}{l}+3\left(\frac{d_a}{l}\right)^2} \tag{16}$$

(b) $M_b = 0$. That is the case which the section B is a pin joint. Substitute θ_b of Eq. (7) for θ_b in Eq. (15) then,

$$M_{ab} = \frac{1.5\frac{d_b}{l}+1.5\frac{d_a d_b}{l^2}}{1.5+3\frac{d_a}{l}+1.5\left(\frac{d_a}{l}\right)^2} = \frac{\frac{d_b}{l}\left(1+\frac{d_a}{l}\right)}{1+2\frac{d_a}{l}+\left(\frac{d_a}{l}\right)^2} \tag{17}$$

(c) $M_a = 0$. That is member built-in at one end, pin joint at Section A. From Eq. (9) and (15) we have:

$$M_{ab} = \frac{\left(1+3\frac{d_b}{l}\right)-\left[4+6\frac{d_a}{l}+6\frac{d_b}{l}+9\frac{d_a d_b}{l^2}\right]+\left[3\frac{d_a}{l}+6\frac{d_a d_b}{l^2}\right]}{2+6\frac{d_a}{l}+6\left(\frac{d_a}{l}\right)^2-\left[2+9\left(\frac{d_a}{l}\right)+9\left(\frac{d_a}{l}\right)^2\right]} = \frac{1+\frac{d_a+d_b}{l}+\frac{d_a d_b}{l^2}}{\frac{d_a}{l}\left[1+\left(\frac{d_a}{l}\right)\right]} = \frac{1+\frac{d_b}{l}}{\frac{d_a}{l}} \tag{18}$$

(d) $\theta_a = \theta_b$. Eq. (15) can be changed into the following equation.

$$M_{ab} = \frac{\left[3+6\frac{d_b}{l}+3\frac{d_a}{l}+6\frac{d_a d_b}{l^2}\right]}{\left[3+9\frac{d_a}{l}+6\left(\frac{d_a}{l}\right)^2\right]} = \frac{1+2\left(\frac{d_b}{l}\right)}{1+2\left(\frac{d_a}{l}\right)} \tag{19}$$

(e) $\theta_a = -\theta_b$. Then,

$$M_{ab} = \frac{1+3\frac{d_b}{l}-2-3\frac{d_b}{l}+3\frac{d_a}{l}+6\frac{d_a d_b}{l^2}}{1+3\frac{d_a}{l}+6\left(\frac{d_a}{l}\right)^2} = \frac{-1+3\frac{d_a}{l}+6\frac{d_a d_b}{l^2}}{1+3\frac{d_a}{l}+6\left(\frac{d_a}{l}\right)^2} \tag{20}$$

MATRIX FORMULATION OF SLOPE DEFLECTION EQUATIONS[a]

Discussion by Ming L. Pei

MING L. PEI,[1] A. M. ASCE.—Professor Wang has presented a very interesting paper on how to formulate a rigid frame problem for solution on electronic digital computers. The timeliness and the profession's interest in the subject is indicated by the fact that another paper[1] also appeared in the same issue of the Journal of the Structural Division.

The object of a rigid frame analysis is to determine the unknown joint rotation (and side-sway) matrix X due to a load matrix P. The basic relationship between these two quantities is the matrix equation,

$$K X = P \tag{11}$$

where K is designated by the author as the "spring matrix". The joint rotation matrix X is obtained by the equation,

$$X = K^{-1} P \tag{12}$$

Since the inversion of matrix K is to be done on the computer, the problem is reduced to the determination of matrix K. The author obtains K by two matrix multiplications as follows:

$$K = A S A^{T} \tag{13}$$

where S is designated by the author as the "stiffness matrix". The size of matrix A and S, as used by the author, is larger than that of matrix K. The computation of K by Eq. (13) for large rigid frames may require considerable machine time. The purpose of this discussion is to show that, in many instances, the matrix K can be obtained directly from stiffness matrices of members of the rigid frame by scalar additions.

The slope-deflection equations of a member with constant moment of inertia and without side-sway are,

$$M_1 = 4EK \phi_1 + 2EK \phi_2$$

$$M_2 = 2EK \phi_1 + 4EK \phi_2$$

The same equations can be stated in matrix as

$$M = S \phi \tag{14}$$

---

a. Proc. Paper 1819, October, 1958, by Chu-kia Wang.

1. New York, N. Y.

where,

$$M = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \qquad \phi = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} \tag{15}$$

$$S = \begin{bmatrix} 4EK & 2EK \\ 2EK & 4EK \end{bmatrix}$$

The matrix S is the stiffness matrix for a single member. Consider now author's Example No. 1 which is a continuous beam. If we use the notation shown in Fig. 6a, the stiffness matrices for the three members are:

$$S_{12} = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix} \quad S_{34} = \begin{bmatrix} 5/3 & 5/6 \\ 5/6 & 5/3 \end{bmatrix} \quad S_{56} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

If we assemble the stiffness matrices for the members into one large matrix, we have author's matrix S

$$S = \begin{bmatrix} 1 & 1/2 & & & & \\ 1/2 & 1 & & & & \\ & & 5/3 & 5/6 & & \\ & & 5/6 & 5/3 & & \\ & & & & & \end{bmatrix}$$

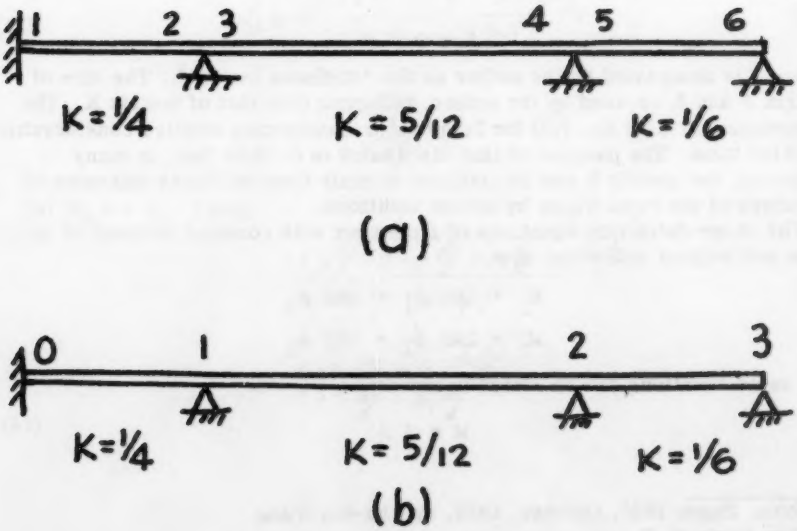

Fig. 6 Example No. 1

$$S = \begin{bmatrix} & & 2/3 & 1/3 \\ & & 1/3 & 2/3 \end{bmatrix}$$

This is the same as that given in Table 1.3. The author then obtained K from S by means of Eq. (3). If, however, we begin with the notation shown in Fig. 6b, the stiffness matrices for the members will be as follows:

$$S_{01} = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix} \qquad S_{12} = \begin{bmatrix} 5/3 & 5/6 \\ 5/6 & 5/3 \end{bmatrix} \qquad S_{23} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

Combining these into one matrix, one obtains at once author's matrix K.

$$K = \begin{bmatrix} 1 & 1/2 & & \\ 1/2 & 1+5/3 & 5/6 & \\ & 5/6 & 5/3+2/3 & 1/3 \\ & & 1/3 & 2/3 \end{bmatrix} = \begin{bmatrix} 1 & 1/2 & & \\ 1/2 & 8/3 & 5/6 & \\ & 5/6 & 7/3 & 1/3 \\ & & 1/3 & 2/3 \end{bmatrix}$$

The simplification is due to the assignment of one letter to each joint, as in Fig. 6b, instead of one letter to each end of a member, as shown in Fig. 6a. Along the diagonal of matrix K, the value of each element is $\Sigma 4EK$, which is the stiffness of the joint. The additions are easy to carry out, either manually or by a computer, than the matrix multiplications $A\,S\,A^T$.

The "spring matrix" K has been called the stiffness matrix of the structure by many authors,[1,2,3] which seems to be a more appropriate term.

The slope deflection equations for a member with constant moment of inertia and side-sway δ (see Fig. 7) are,

$$\begin{aligned} M_1 &= 4EK\,\phi_1 + 2EK\,\phi_2 - 6EK/L\;\delta_3 \\ M_2 &= 2EK\,\phi_1 + 4EK\,\phi_2 - 6EK/L\;\delta_3 \\ P_3 &= -6EK/L\,\phi_1 - 6EK/L\,\phi_2 + 12EK/L^2\,\delta_3 \end{aligned} \tag{16}$$

Eq. (16) may be written in the matrix form as follows:

$$M = S\,\phi \tag{14}$$

where,

$$M = \begin{bmatrix} M_1 \\ M_2 \\ P_3 \end{bmatrix}, \qquad \phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \delta_3 \end{bmatrix}$$

and the stiffness matrix for uniform member with side-sway S is,

$$S = \begin{bmatrix} 4Ek & 2EK & -6EK/L \\ 2FK & 4EK & -6EK/L \\ -6EK/L & -6EK/L & 12EK/L^2 \end{bmatrix} \tag{15}$$

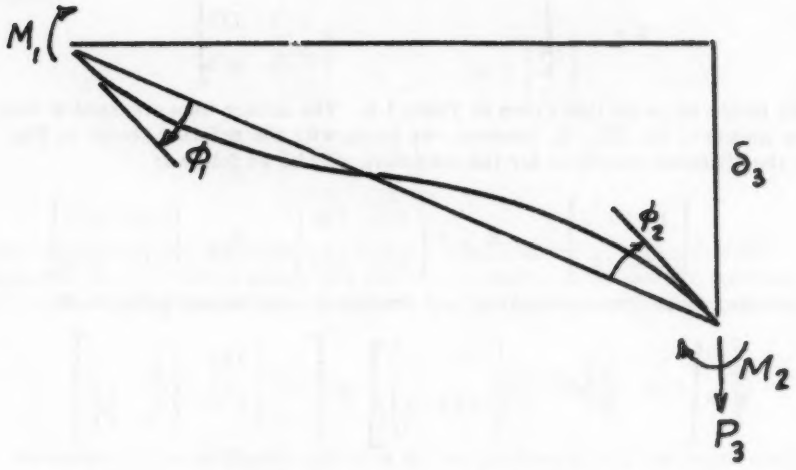

Fig. 7

Based on Eq. (15), and using the notation shown on Fig. 8, the stiffness matrices for author's Example No. 2 can be written at once.

$$S_{013} = \begin{bmatrix} 4/15 & 2/15 & -2/75 \\ 2/15 & 4/15 & -2/75 \\ -2/75 & -2/75 & 4/1125 \end{bmatrix}$$

$$S_{243} = \begin{bmatrix} 2/5 & 1/5 & -3/50 \\ 1/5 & 2/5 & -3/50 \\ -3/50 & -3/50 & 3/250 \end{bmatrix}$$

$$S_{12} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

These matrices are assembled to form matrix K.

$$K = \begin{bmatrix} 4/15 & 2/15 & & -2/75 & \\ 2/15 & 4/15+2/3 & 1/3 & -2/75 & \\ & 1/3 & 2/5+2/3 & -3/50 & 1/5 \\ -2/75 & -2/75 & -3/50 & 4/1125+3/250 & -3/50 \\ & & 1/5 & -3/50 & 2/5 \end{bmatrix}$$

$$= \begin{bmatrix} 4/15 & 2/15 & & -2/75 & \\ 2/15 & 14/15 & 1/3 & -2/75 & \\ & 1/3 & 16/15 & -3/50 & 1/5 \\ -2/75 & -2/75 & -3/50 & 7/450 & -3/50 \\ & & 1/5 & -3/50 & 2/5 \end{bmatrix}$$

Thus it seems that in many instances nothing is gained by using the formula $K = A\,S\,A^T$ to compute K. It can be obtained simply by adding all of the stiffness matrices of the members, provided of course that one number only is assigned to each joint rotation or side-sway displacement, as shown in

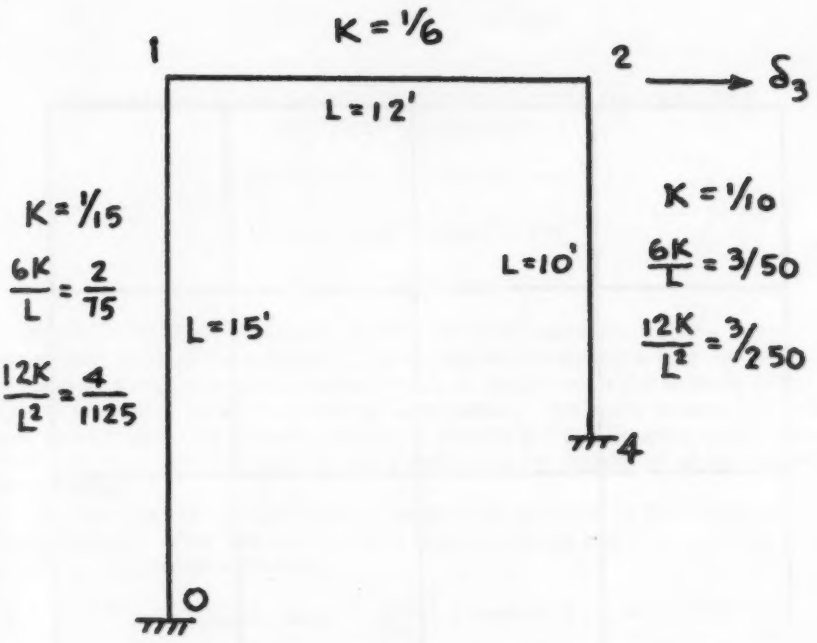

Fig. 8 Example No.2

Figs. 6b and 8. When the structure is not too large, the additional computer time required may be inconsequential, and Professor Wang's method has great merit because it is straightforward and methodical. For larger structures, one has to be more careful. Consider the rigid frame shown in Fig. 9 for example. Using the procedure outlined above, the matrix is of order 35. Using Professor Wang's procedure, the matrix is of order 90. Whether the available computer is capable of handling the larger matrix, and how much computer time is required, should be carefully evaluated.

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1. Archer, J. S., "Digital Computation for Stiffness Matrix Analysis," ASCE Proc. Paper 1814, October, 1958.

2. Kron, G., "Tensorial Analysis and Equivalent Circuits of Elastic Structures," Journ. Franklin Inst., Vol. 238, No. 6, 1944.

3. Levy, S., "Structural Analysis and Influence Coefficients for Delta Wings," J. Aero. Sci., Vol. 20, No. 7, 1953.

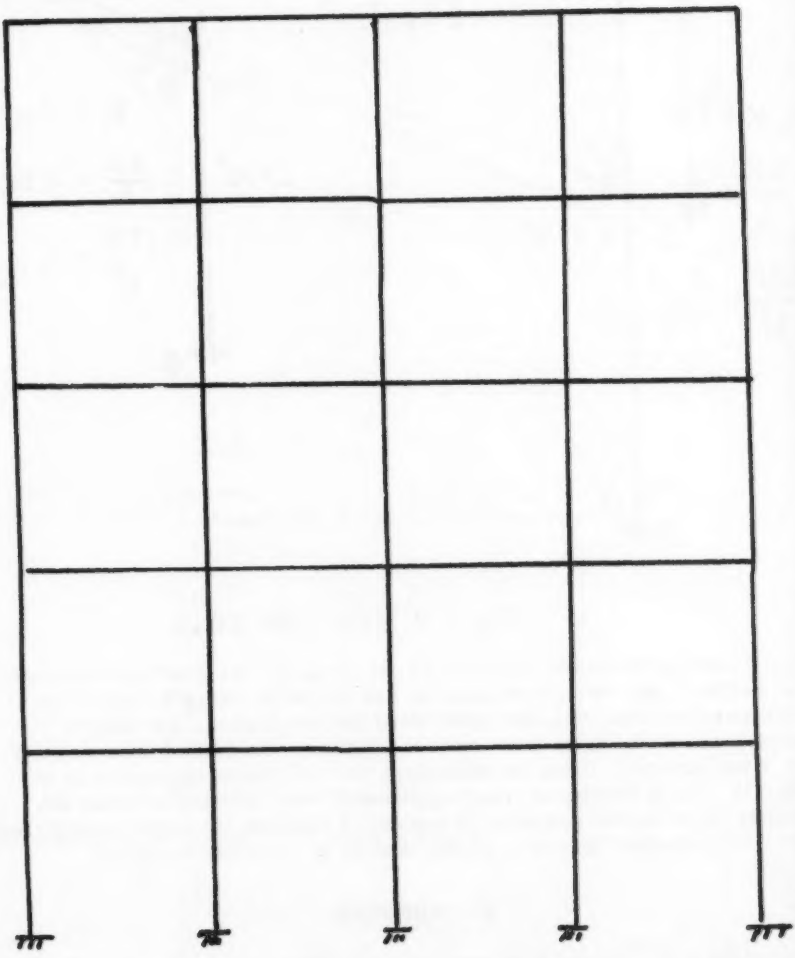

Fig. 9

EQUIVALENT SYSTEMS FOR THE DEFLECTION OF VARIABLE STIFFNESS MEMBERS[a]

Discussion by Perng-Fei Gou

PERNG-FEI GOU.[1]—Messrs. Fertis and Zobel have presented a very interesting method for calculating the deflection of members with variable stiffness. The approximate method which is suggested by the authors is very convenient and accurate in practical application. The exact method which is also submitted by the authors gives real results but the computation is clumsy. Hence we may consider that the exact method is the foundation of the approximate method.

The accuracy of the approximate method can be found in the following computations. (This example is taken from the essay which the authors give.)

By the approximate method,

$$\delta = M_{(\text{on conjugate beam})} = \frac{1}{E_1 I_1}\left[30\times10\times10\times\tfrac{1}{2}\times\tfrac{2}{3} + 30\times14\times17 + \tfrac{1}{2}\times97\times14\left(\tfrac{2}{3}\times14\times10\right) + 127\times36\times42 + \tfrac{1}{2}\times323\times36\left(\tfrac{2}{3}\times36\times24\right)\right]$$

$$= 492{,}260\cdot\frac{1}{E_1 I_1}$$

By the usual method,

$$\delta = \frac{1}{E_1 I_1}\int_0^l \frac{2l x^2}{2(l+3x)}\,dx$$

$$= \frac{2l}{2E_1 I_1}\left[\int_0^l\left(\frac{x^2}{3} - \frac{lx}{9} + \frac{l^2}{27}\right)dx - \frac{l^3}{27}\int_0^l \frac{3\,dx}{3(l+3x)}\right]$$

$$= \frac{2l}{2E_1 I_1}\left[\frac{l^3}{9} - \frac{l^3}{18} + \frac{l^3}{27} - \frac{l^3}{27\times3}\ln(l+3l) + \frac{l^3}{27\times3}\ln(l)\right]$$

$$= \frac{30}{E_1 I_1}\left(\frac{5l^3}{54} - \frac{l^3}{81}\ln 4\right)$$

$$= 496.000\,\frac{1}{E_1 I_1}$$

$$\text{Error} = \frac{3740}{496000} = \frac{1}{133} < 1\%$$

a. Proc. Paper 1820, October, 1958, by D. G. Fertis and E. C. Zobel.

1. Asst., Civ. Eng. Dept., Cheng Kung Univ., Tainan, Taiwan, Free China.

To determine the deflection of a member with variable stiffness the following procedures will be suggested.

1st step. Draw the moment of inertia diagram.
2nd step. Draw the moment diagram.
3rd step. Draw the equivalent moment diagram, i.e., $\frac{\text{M actual at x}}{\text{I at x}}$ diagram.
4th step. Approximate the M_{eq}, curve.
5th step. Use the conjugate beam method or moment area method to determine the deflection.

In actual calculation, the determination of equivalent shear force diagram and equivalent force system which are given by the authors can be omitted.

AMPLIFICATION OF STRESS AND DISPLACEMENT IN GUYED TOWERS[a]

Discussion by Vitelmo Bertero

VITELMO BERTERO,[1] A. M. ASCE.—The author is to be commended for his attempt to offer to the designer simplified equations for the determination of the slack stress in the guys and displacements at guy levels, and a simplified method for solving the problem of amplification of stress and displacements in guyed towers. However, it is the writer's opinion that several points of the method of attack used in the derivation of the simplified equations should be clarified, so that this valuable paper may become more useful to engineers concerned for the first time with this type of structure. To start with the relatively unimportant, it is suggested that a few of the author's equations appear to have been misprinted. The term $\frac{fa}{nE}(\frac{L}{r})^2$ in Eq. (15) should be $\frac{n\,fa}{E}(\frac{L}{r})^2$ if a value of $n = \frac{1}{10}$ is used. On page 14, in eq. for Mb, and Eq. (20) for Ma the term Δe should be ΔH. Also, in page 13, 15 and 16 the term $\frac{1}{r}$ should be $\frac{L}{r}$.

The statement that follows Eq. (10) is not clear, because in order for Eqs. (9) and (10) to yield the same value, the total cross sectional area of the six guys for double guys should be equal to the total area of the three guys for single guys, and not to twice this area.

The writer agrees that the method developed by the author for determining the slack stress in the guy and horizontal displacements due to wind load is very simple and easy to perform. However, it is felt that this method is based on a set of working formulae which does not offer satisfactory accuracy for tall towers.

The relationship between the live-load force, the initial tension and the slack stress in the guys represented in the chart of Fig. 4 was obtained from simplified equations, where the effect of the sag of the cable and the effect of wind pressure along the length of the guy, were neglected. The writer believes that these effects should be included and, in order to show the relative importance of each one, they will be separately considered in the discussion which follows.

In a study made by M. J. Holley, Jr. and the writer a few years ago,(1) it was found that if the effect of the wind along the guy is neglected, the following expression relating upper end displacement with the change in guy tension of an individual guy, proves to be very useful in the study of the behaviour of a guy group.

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a. Proc. Paper 1821, October, 1958, by Robert S. Rowe.

1. Lecturer in Civ. Eng., Univ. of California, Berkeley, Calif.

$$\Delta D = \frac{D}{E}\cdot\sec^2\gamma\cdot\Delta f_a + \frac{\rho^2 D^3 \sec^2\gamma}{24 f_a^2}\left[1 - \frac{1}{\left(1+\frac{\Delta f_a}{f_a}\right)^2}\right] \qquad (a)$$

where, as it is shown in Fig. A,

ΔD = horizontal displacement of the upper end of the guy

D = initial horizontal projection of the guy.

E = modulus of elasticity of cable.

γ = inclination of the chord.

f_a = mid-length unit stress of the guy.

Δf_a = change in the mid-length unit stress f_a of the guy.

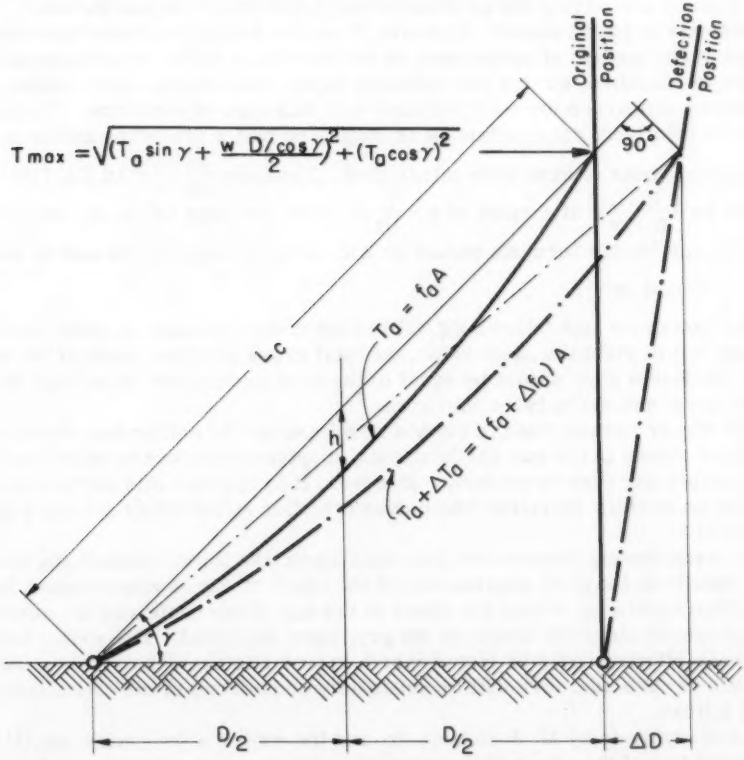

Fig. A

Sketch showing displacement of upper end of a single guy.

p = density of the guy = $\frac{\text{weight per foot}}{\text{metallic area}}$

The first term of Eq. (a) represents the horizontal displacement that would result if the cable were considered as a tie rod; i.e., a straight two force member. Thus the second term of this equation may be considered as a correction to the tie rod displacement, to account for the effect of sag. This correction is approximated, but it is believed to be of satisfactory accuracy for application to the range of sag ratios ($\frac{h}{D}$) encountered in properly designed guyed towers.

Expression (a) developed for an individual guy can be plotted. By consideration of the compatibility of displacement of windward and leeward guys, and the required conditions of equilibrium, the displacement versus change in tension of the individual guys can be combined to obtain curves which define the resistance function of a group of guys; that is, the relation between displacement of the group and net force which the group exerts on the tower. If this is done for the case classified by the author as Wind I, Fig. B shows that if the horizontal displacement of the tower at the guy connection is called δ, then,

$$\begin{aligned} \Delta D_1 &= \delta_I \\ \Delta D_2 &= \Delta D_3 = -0.5\,\delta_I \end{aligned} \tag{b}$$

and

$$H_I = \left(\Delta f_{a_1} - \Delta f_{a_2}\right) \cdot A \cdot \cos\gamma \tag{c}$$

Eq. (a) can be written as

$$\Delta D = \left(I + II\right) \sec^2\gamma \tag{a1}$$

where

$$I = m\,\Delta f_a \quad ; \quad m = \frac{D}{E}$$

$$II = \alpha\left(1-\beta^2\right) \quad \text{where} \quad \alpha = \frac{p^2 D^3}{24 f_a^2} \quad \text{and} \quad \beta = \left(\frac{1}{1+\frac{\Delta f_a}{f_a}}\right)$$

From (b) and using (a^1)

$$\begin{aligned} \delta_I &= \Delta D_1 = \left(I + II\right) \sec^2\gamma \\ \delta_{II} &= -2\,\Delta D_2 = -2\,\Delta D_3 = 2\left(I + II\right) \sec^2\gamma \end{aligned} \tag{b1}$$

Using Eq. (b^1) the curves of Δfa_1 and $-\Delta fa_2$ versus δ_1 for any particular case can be easily obtained as will be illustrated by considering the set of single guys 1-3/8" diameter at the third guy level of the typical tower shown by the author in Fig. 1. The significant guy data for this example are as follows.

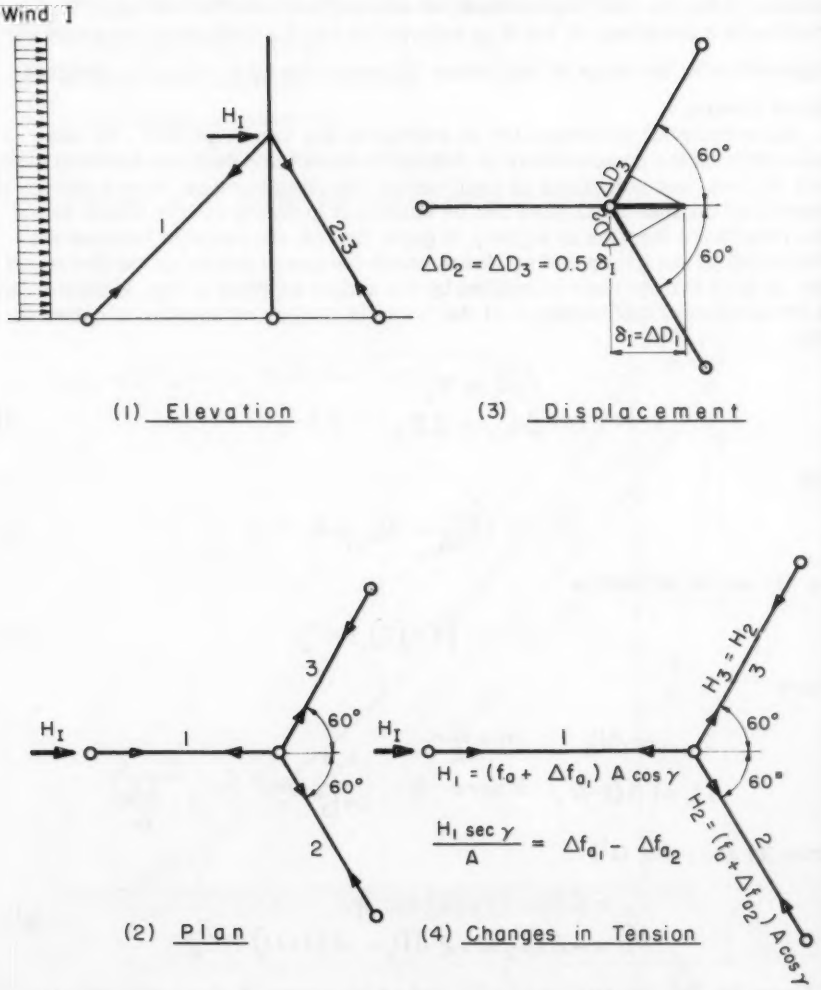

Fig. B

Sketch showing displacement and changes in tension for a three guy connection due Wind I.

From Reference (2)

$$\begin{cases} \text{weight } w = 3.94 \ \frac{lbs}{ft} \\ E = 25 \cdot 10^6 \ \frac{lbs}{in^2} \\ A = 1.135 \ in^2 \\ B.S. = 232{,}000 \ lbs \ (\text{breaking strength}) \end{cases}$$

$$\rho = \frac{w}{A} = 3.46 \ \frac{lbs}{in^2 ft} \quad \therefore \quad \rho^2 = 12 \left(\frac{lbs}{in^2 ft}\right)^2$$

For cable 1 $D = +900$ ft.

For cables 2 and 3 $D = -900$ ft.

Considering for the initial mid-length tension the value selected by the author; i.e., P_i = 0.2 B.S. = 46,400 lbs. the corresponding initial unit tension would be

$$f_a = \frac{46{,}400}{1.135 \ in^2} = 40{,}900 \ \frac{lbs}{in^2}$$

and for this value of fa, the variation of δ with changes Δfa_1, and $-\Delta fa_2$ and $-\Delta fa_3$ can be computed as indicated in Table 1.

The values of Δfa_1, and $-\Delta fa_2 = -\Delta fa_3$ were plotted versus the obtained values of δ_I, as shown on Fig. C. The curves in the right quadrant are drawn for Wind I. Selecting a factor of safety F.S = 2.5, as the author suggested, the maximum allowable tension in the cable would be

$$T_{allowable} = \frac{B.S.}{F.S.} = \frac{232{,}000 \ lbs}{2.5} = 92{,}800 \ lbs.$$

For an initial mid-length tension of Ta = Pi = 46,400 lbs, the maximum initial tension in the cable, which occurs at the upper end, may be computed as follows:

$$T_{\substack{initial \\ max.}} = \sqrt{\left(T_a \sin\beta + \frac{w D/(\cos\beta)}{2}\right)^2 + \left(T_a \cos\beta\right)^2}$$

$$T_{\substack{initial \\ max.}} = \sqrt{\left(46{,}400 \cdot 0.707 + \frac{3.94 \cdot 900}{2 \cdot 0.707}\right)^2 + \left(46{,}400 \times 0.707\right)^2} = 48{,}500 \ lbs.$$

Accordingly it becomes evident, that the allowable value of the wind load H_I is that which produces a δ_I at the third guy level consistent with a change in the tension of guy 1 ΔT_1 = 92,800 - 48,500 = 44,300 lbs. This corresponds to $\Delta fa_1 = \frac{44{,}300 \text{ lbs}}{1.135 \text{ in}^2} = 39{,}000 \ \frac{\text{lbs}}{\text{in}^2}$. For this value of Δfa_1 the magnitude of δ_I can be computed using Eq. (b[1]) and is δ_I = 3.12 ft. This value may be also read directly from the curve of Fig. C. From this same figure, it can be seen that for δ_I = 3.12 ft, cables 2 and 3 undergo a change in tension $\Delta fa_2 = \Delta fa_3 = -14{,}000 \ \frac{\text{lbs}}{\text{in}^2}$. Consequently the allowable wind load H_I can be obtained using Eq. (c)

$$H_I = \left[39{,}000 - (-14{,}000)\right] 1.135 \cdot 0.707 = 42{,}600 \ \frac{lbs.}{in^2}$$

which in terms of the breaking strength of the cable becomes

TABLE I

COMPUTATION OF σ BASED ON

$\sigma = (I+II)\sec^2\gamma$ where $\begin{cases} I = m \cdot \Delta f_a = \frac{D}{E}\Delta f_a = \frac{3.6 \text{ ft}}{10^5 \text{ lb/in}^2} \cdot \Delta f_a \\ II = \alpha(1-\beta^2) \end{cases}$ where $\alpha = \frac{12}{24} \cdot \frac{900^3}{40{,}900^2} = 0.218$

for $f_a = 40{,}900 \frac{\text{lbs}}{\text{in}^2}$

Δf_a Wind I Δf_{a_1} / Wind II $\Delta f_{a_2} = \Delta f_{a_3}$ $\frac{\text{lbs}}{\text{in}^2}$	$I = \frac{3.6}{10^5} \cdot \Delta f_a$ (ft)	$\frac{\Delta f_a}{f_a}$	$\beta = \frac{1}{1+\frac{\Delta f_a}{f_a}}$	$1-\beta^2$	II $0.218(1-\beta^2)$ (ft)	$I+II$ (ft)	For Wind I $\sigma_I = (I+II)2$ (ft)	For Wind II $\sigma_{II} = (I+II)4$ (ft)
5,000	0.180	0.122	0.890	0.210	0.046	0.226	0.452	0.904
10,000	0.360	0.244	0.803	0.357	0.078	0.438	0.876	1.752
15,000	0.540	0.367	0.731	0.468	0.102	0.642	1.284	2.568
20,000	0.720	0.489	0.671	0.550	0.120	0.840	1.680	3.360
25,000	0.900	0.611	0.620	0.615	0.134	1.034	2.068	4.136
30,000	1.080	0.733	0.576	0.669	0.146	1.226	2.452	4.904
35,000	1.260	0.855	0.539	0.710	0.155	1.415	2.830	5.660
40,000	1.440	0.978	0.505	0.745	0.162	1.602	3.204	6.408
45,000	1.620	1.100	0.476	0.774	0.168	1.788	3.576	7.152
50,000	1.800	1.222	0.450	0.798	0.174	1.974	3.948	7.896

ST 5

Table I, Continuation

Δf_a Wind I $\Delta f_{a_2}=\Delta f_{a_3}$ / Wind II Δf_{a_1} (Lbs./in²)	$I = \frac{3.6}{10^5}\Delta f_a$ (ft)	$\frac{\Delta f_a}{f_a}$	$\beta = \frac{1}{1+\frac{\Delta f_a}{f_a}}$	$1-\beta^2$	$II = .218(1-\beta^2)$ (ft)	$I+II$ (ft)	For Wind I $d_I=(I+II).4$ (ft)	For Wind II $d_{II}=(I+II).2$ (ft)
- 5,000	-0.180	-0.122	1.138	-0.295	-0.064	-0.244	-0.976	-0.488
-10,000	-0.360	-0.244	1.321	-0.745	-0.161	-0.521	-2.084	-1.042
-15,000	-0.540	-0.367	1.578	-1.485	-0.325	-0.865	-3.460	-1.730
-17,500	-0.630	-0.428	1.746	-2.050	-0.448	-1.078	-4.312	-2.156
-20,000	-0.720	-0.489	1.955	-2.830	-0.616	-1.336	-5.344	-2.672
-22,500	-0.810	-0.550	2.220	-3.930	-0.857	-1.667	-6.668	-3.334
-25,000	-0.900	-0.611	2.565	-5.600	-1.221	-2.121	-8.484	-4.242
-26,000	-0.936	-0.635	2.740	-6.500	-1.435	-2.371	-9.484	-4.742
-27,000	-0.972	-0.660	2.940	-7.650	-1.670	-2.642	-10.564	-5.282
-28,000	-1.008	-0.684	3.160	-9.000	-1.960	-2.970	-11.880	-5.940
-29,000	-1.044	-0.709	3.46	-10.8	-2.360	-3.404	-13.616	-6.808

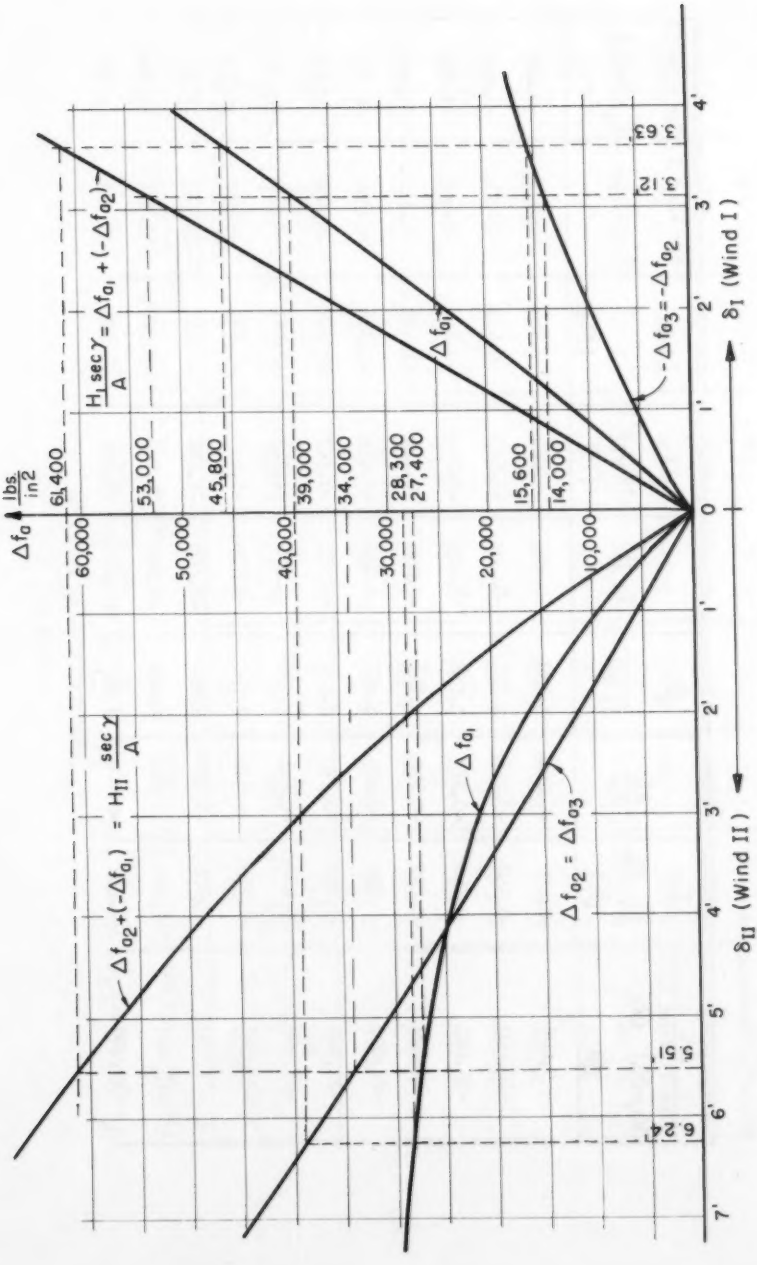

Fig. C. Displacement δ versus change in tension Δf_a

$$H_I = \frac{42{,}600}{232{,}000}\ B.S = 0.184\ B.S$$

The mid-length tension in the cables 2 and 3 would be

$$T_{a_2} = T_{a_3} = (40{,}900 - 14{,}000).1.135 = 30{,}600 lbs = 0.132 B.S.$$

This value corresponds to what the author designated as slack force P_{SI}.

The increase in the vertical force on the mast would be

$$\Delta V = (39{,}000 - 2.14{,}000).1.135.0.707 = 8{,}850 lbs.$$

or

$$\Delta V = 0.0835 V.$$

where V is the initial vertical force on the mast, which is equal to

$$V = 3\left(40{,}900.1135.0.707 + \frac{1}{2}.3.94.\frac{900}{0.707}\right) = 106{,}000\ lbs.$$

If the case classified by the author as WIND II is considered, it is evident that since wind direction is reversed from that shown in Fig. B, the following equations result

$$\Delta D_1 = -\delta_{II}$$

$$\Delta D_2 = \Delta D_3 = 0.5\,\delta_{II} \qquad (e)$$

$$H_{II} = (\Delta f_{a_2} - \Delta f_{a_1}).A.\cos\gamma \qquad (f)$$

using (a[1])

$$\delta_{II} = -\Delta D_1 = (I + II)\sec^2\gamma$$

$$\delta_{II} = 2\Delta D_2 = 2\Delta D_3 = 2(I+II)\sec^2\gamma \qquad (e^1)$$

Taking advantage of the values already computed in Table 1 for WIND I, the curves shown on the left quadrant of Fig. C can be plotted. Using these curves, it can be shown that the allowable wind load H_{II} would be

$$H_{II} = (39{,}000 + 28{,}300)\,1{,}135.0.707 = 54{,}000 lbs.$$

and the required horizontal displacement to produce this net force is

$$\delta_{II} = 6.24\ ft.$$

Comparing H_I and H_{II} it is clear, as it might be expected, that WIND I controls; and consequently, the maximum allowable wind load would be $H_I = 0.184$ B.S. instead of 0.30 B.S. $\sin\alpha = 0.212$ B.S. as found by the author.

If the value of $H_I = 0.30$ B.S. $\sin\alpha$ is considered, according to the author's Eq. (8) the magnitude of the corresponding horizontal displacement would be

$$\Delta H = \frac{0.2\ B.S.}{\sin\alpha}\frac{L_c}{A_c E_c} = \frac{0.2.232{,}000.900/0.707}{0.707.1135.25.10^6} = 2.94\ ft.$$

and, this value is the same for both WIND I and WIND II. However, when the sag effect is considered, using the Eq. (a) suggested by the writer, the following more accurate values result:

WIND I

According Eq. (c), the total change in unit stress required by $H_I = 0.3$ B.S. sin α would be $\Delta fa_1 - \Delta fa_2 = \frac{0.3.232{,}000 \times 0.707}{1.135 \times 0.707} = 61{,}400 \frac{lbs}{in^2}$. Considering this value and using the curves of Fig. C (right quadrant), it can be seen that $\delta_I = 3{,}63$ ft and

$$\Delta f_{a_1} = 45{,}800 \frac{lbs}{in^2}$$
$$\Delta f_{a_2} = \Delta f_{a_3} = -15{,}600 \frac{lbs}{in^2}$$

These changes in the unit stress of the guys originate an increase in the vertical force on the mast, which can be computed as follows:

$$\Delta V = (45{,}800 - 2.15{,}600) 1.135 . 0.707 = 11{,}700 lbs.$$

or

$$\Delta V = \frac{11{,}700}{106{,}000} V = 0.11 V$$

The forces in cable 2 and 3, which correspond to what the author called slack force P_{s_I}, would be equal to $\frac{(40{,}900 - 15{,}600)}{40{,}900}$ 0.2 B.S. = 0.124 B.S.

WIND II

According Eq. (f) for $H_{II} = 0.30$ B.S. sin α, the total change in unit stress would be $\Delta fa_2 - \Delta fa_1 = 61{,}400 \frac{lbs}{in^2}$. Using the curves on the left quadrant of Fig. C, the following values can be obtained:

$\delta_{II} = 5.51$ ft.

$$\Delta f_{a_1} = -27{,}400 \frac{lbs}{in^2}$$
$$\Delta f_{a_2} = \Delta f_{a_3} = 34{,}000 \frac{lbs}{in^2}$$

Therefore $\Delta V = (2 \times 34{,}000 - 27{,}400) \times 1.135 \times 0.707 = 32.700$ lbs. or $\Delta V = 0.308$ V and the force in cable 1, which correspond to what the author designated as $P_{s_{II}}$ would be equal to $\frac{(40{,}900 - 27{,}400) \times 0.2 \text{ B.S.}}{40{,}900} = 0.066$ B.S.

Table 2 summarizes the results obtained applying the simplified method presented by the author and those obtained by including the sag effect according to the method suggested by the writer.

The considerable discrepancies between the results given in Table II clearly indicate the importance of the sag effect. The simplified equations developed by the author neglecting this effect not only yield a larger value for the allowable wind load but also give displacements which are considerably smaller than those to be expected, especially when WIND II is considered. While the author assumes that there is no change in the vertical force on the mast, the consideration of the sag effect shows an increase of more than 30% of the initial vertical force.

Regarding the influence of the wind pressure acting on a guy, there are two effects or modifications that have to be considered. First, the force which the guy exerts on the undisplaced tower is modified. Second, the relations between guy tension and upper end displacement are modified. To account for both of these effects in a rigorous way complicates the analysis. Practical approximation to these effects were suggested in various forms.(1,3,4)

Due to the lack of information about the exact wind pressure used by the author in computing the 200 pounds per foot of tower, the writer could not evaluate with accuracy the effect that the consideration of the wind on the guys would have in the example discussed. However, in order to illustrate the importance of this effect, a new analysis of the behavior of the third guy level group was carried out including the effect of a wind pressure of 50 pounds per square foot acting on the guys. This analysis was done according to the method suggested in reference (1), and the following results were obtained:

WIND I.—The allowable wind load on the tower H_I is reduced to 0.15 B.S. sin α, the corresponding horizontal displacement $\Delta H_I = 2.82$ ft. The initial vertical force V on the mast experiences an increase ΔV of 35.4% of V.

When an allowable wind load $H_I = 0.30$ B.S. sin α as obtained by the author was considered, the maximum tension force in the windward guy became equal to 0.53 B.S. and the corresponding horizontal displacement equal to 4.85 ft. The increase in vertical load ΔV becomes 48% of the initial V.

WIND II.—For $H_{II} = 0.30$ B.S. sin α, ΔH_{II} became equal to 3.75 ft. and the initial vertical force on the mast V underwent an increase of 40% of V.

If the above results are compared with those obtained by the writer and given in Table 2, the importance of the effect of wind pressure along the guy becomes evident. Therefore, it is clear that neither the sag effect nor the effect of wind on guys can be neglected without introducing considerable inaccuracy. Since in all cases the author's method yields values that are non-conservative, it is evident that this method should be used cautiously.

With the presentation of the amplification charts for stress and displacement, the author has made a very practical contribution to the engineers interested in the design of guyed towers.

As it was demonstrated in reference (1) and pointed out in the discussion,(5) quite frequently the tower geometry and anticipated leg stress associated with tower axial load are such as to render the use of beam column theory unnecessary.

The author's charts of Fig. 5 and 6 can be used with advantage to determine whether beam column theory needs to be applied. Certainly the trend toward high strength alloy steel with accompanying high values f_a, would seem to imply that the amplification of stress will be increasingly important, particularly for very large towers. Moreover, if design is based on ultimate load basis (and the writer believes this to be highly desirable), the larger values of the unit stress associated with axial force occurring at ultimate load may well require the application of beam-column theory.

Referring to Example 1 selected by the author to illustrate the use of the Amplification Chart, it should perhaps be pointed out that according to the data given in Fig. 1, the value of the moment in the span L = 238 feet to be used in Eq. (18) should be 1,316$^{k'}$ and not 1,868$^{k'}$. Consequently, the resulting stress f_b and its amplification would be smaller than those obtained by the author.

TABLE 2

RESULTS FOR THE THIRD SET OF GUYS ASSUMING $P_t = 0.20\ B.S.$

Results obtained for / According	WIND LOAD I				WIND LOAD II $= 0.30\ B.S.\sin\alpha$		
	H_I Allowable Wind Load	$B.S._I$ Slack Force	ΔH or d_I Maximum Horizontal Displacement (ft.)	ΔV Change in Vertical Force on Mast in % of Initial Vertical Force V	$B.S._{II}$ Slack Force	ΔH or d_{II} Maximum Horizontal Displacement (ft.)	ΔV Change in Vertical Force on Mast in % of Initial Vertical Force V
Author's Method	$0.30\ B.S.\sin\alpha$	$0.10\ B.S.$	2.94	0.00	0.00	2.94	0.00
More Accurate Method Including Sag Effect	$0.26\ B.S.\sin\alpha$	$0.13\ B.S.$	3.12	8.35			
	Using $0.30\ B.S.\sin\alpha$	$0.12\ B.S.$	3.63	11.10	$0.06\ B.S.$	5.51	30.80

According to the arbitrary limitations for the different ranges of the structural action suggested on page 16, the results obtained for the amplification of displacement and stress in the case of example 1 would seem to indicate that the tower is in the ordinary range and not in the flexible one as stated on page 13.

The writer investigated(1) the effect of guy tension maladjustments (due either to improper installation or to service conditions) and agrees with the author concerning the danger of guy lengthening and shortening. However, it is felt that the formulae developed and used by the author are not reliable, and the following facts should be mentioned for clarification.

The author's statement that "equation (11) is derived for guy lengthening or guy shortening, Δe, at a given guy level" is not clear. It seems that, even if the sag effect is neglected (as suggested by the author) and a set of three single guys is assumed, expression (11) determines the horizontal displacement ΔH that the upper end connection undergoes when the initial tension in only one guy is increased or decreased by $\Delta e \cdot \frac{AgEg}{Lg}$. Using the relationships given in Fig. B-3 and B-4, it can be shown that in order to produce this change in tension it would be necessary to shorten or to lengthen the guy in $3\Delta e$.

Eq. (11) was derived neglecting the sag effect but including the effect of the vertical displacement which the upper end connection of the guy undergoes as a consequence of the variation in the compression force on the tower. As it was pointed out before, the writer believes that the sag effect cannot be neglected. On the other hand, he felt that the effect of vertical displacement of the tower is usually small and it may be neglected, as will be illustrated in the following discussion of the results obtained by the author in his Example 2.

Assuming a change in tension of 45,000 lbs., the author obtained a Δe equal to 3.73 in. and $\Delta H = 10.60$ in., where the effect of the vertical displacement is only 1.2%.

It should perhaps be pointed out that the above value of 1.2% seems to be not quite correct because, in the first place, a factor of 3 was omitted; and secondly, the author used for a guy of 1-5/8 inch diameter a value of $AgEg = 2.05 \times 30.10^6$ lbs. A more realistic value would be obtained by using the metallic area of the guy, $Ag = 1.59$ in^2, and the modulus of elasticity Eg related to this metallic area, which for commonly used Prestressed Galvanized Bridge Strand is approximately $25 \times 10^6 \frac{\text{lbs}}{\text{in}^2}$. If these values are used in the author's formulae, Δe becomes equal to 5.76 in. and

$$\Delta H = 5.76 \cdot \frac{2}{0.707}(1 + 0.02) = 16.65 \text{ in.}$$

It can be seen that the influence of the vertical displacement of the tower is still only 2% and consequently may be neglected.

The importance of the sag effect can be shown by comparing the results obtained by using the author's Eq. (11) with those determined by the application of Eq. (a) suggested by the writer. While Eq. (11) will yield identical values for a certain change in tension regardless if this change is an increase or decrease and regardless of the magnitude of the initial tension, the use of Eq. (a) will show considerable difference. As an example, assuming that the initial tension in the lower set of guys considered by the author was 60,000 lbs. and if the tension in one guy is increased by 45,000 lbs, the upper connection

would undergo a $\Delta H = 16.5$ in. (neglecting effect of vertical displacement). On the other hand, if the tension is decreased by 45,000 lbs, the ΔH becomes equal to 23.2 in.

Referring to the displaced position of the tower axis indicated in Fig. 7, it is not clear to the writer how this could be obtained. If this was the result of lengthening only one guy of G_1, as suggested by the author, it seems that the guy group G_2 had to move to the right from the original position and not to the left. Moreover, the horizontal displacement ΔH of the tower at the first guy level G_1 was computed from Eq. (11) under the assumption that the tower could move freely. This is not the case illustrated in Fig. 7, because the guy groups G_2 and G_3 will restrain the movement of the tower and, consequently, the actual ΔH at G_1 would be somewhat less.

Another point which is not quite clear to the writer is how the value of $P_i = 218{,}000$ lbs. used in Eq. (20) was obtained. It would be desirable to have the author state what initial guy tension was used for each guy level in this example because it seems that the values given in page 2 cannot be used in this case since at first guy level there exists an initial tension per guy of only 31.7 kips and consequently would be impossible to obtain a decrease in tension of 45.0 kips by lengthening the cable.

In determining the actual total stress and horizontal displacement at guy level G_1, an amplification factor of $\Phi = 1.7$ was used. This value was obtained directly from the chart of Fig. 5, using an $\frac{L}{r} = 120$. The validity of this procedure for the case under consideration seems to be doubtful due to the reasons which follow.

As it is indicated on page 8, "the amplification chart is based on a computed total stress of 20 kips per sq. in. at the center of a tower section between points of inflection which supports a uniformly distributed load throughout the tower height." In the case of Example 2, not only the tower does not support any uniformly distributed load and the bending stress is computed not at the center, but what is even more important, between the points of inflection there exists guy group G_1, which will restrain the amplification of the horizontal displacement ΔH at guy level G_1.

The effectiveness of guy group G_1 in restraining the amplification of ΔH will depend upon the tension which exists in each guy. However, it is evident that not only the amplification factor Φ would be smaller than 1.7, but also the critical buckling stress would be larger than the value of 20,600 psi. determined by the author using Eq. (19). A more accurate approach to the problem would be to consider the portion of the tower between the inflection points as a continuous bar on three supports which are not on a straight line and the middle one of which is elastic.

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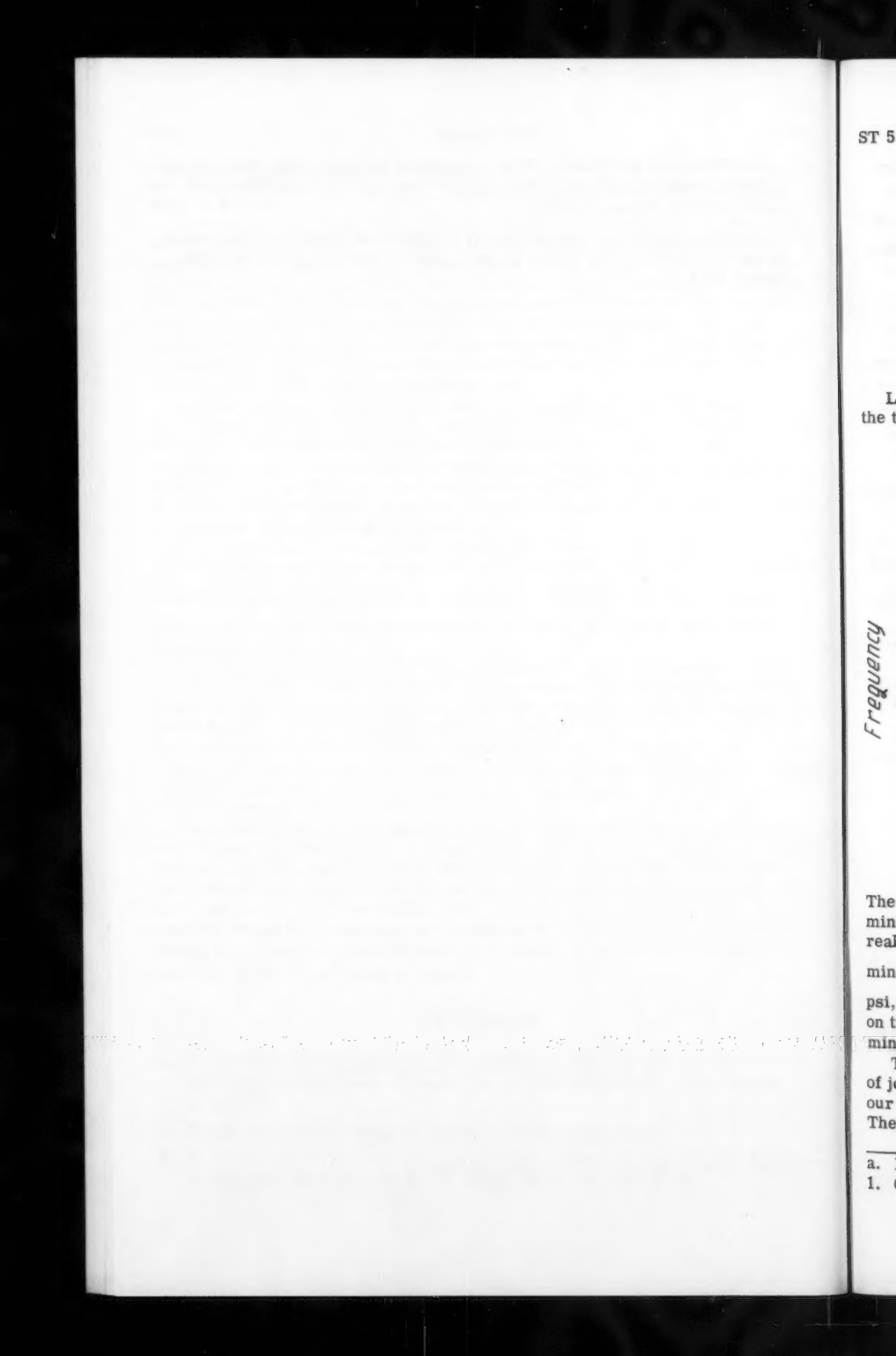
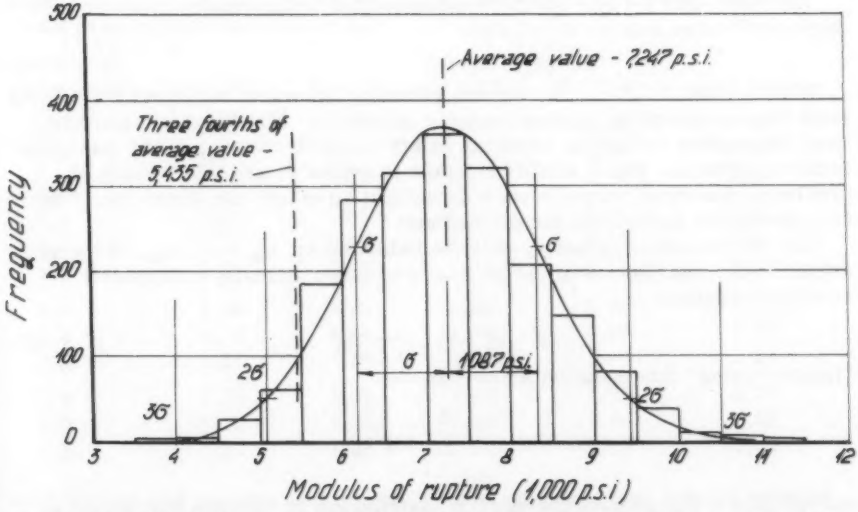


THE FACTOR OF SAFETY IN DESIGN OF TIMBER STRUCTURES[a]

Discussions by L. Cizek and Milos Vorlicek

L. CIZEK.[1]—The writer has prepared the frequency distribution (Fig. 1) by the theoretical curve of Gauss with the maximum ordinate

$$\max y = 0{,}399 \frac{\Delta}{\delta} \cdot n = 0{,}399 \frac{500}{1087} \cdot 2038 = 374 .$$



The standard deviation is δ = 1087 psi and the theoretical minimum strength min k = M - 3δ = 7247 - 3 x 1087 = 3986 psi corresponds quite well with the real minimum strength. The long-time loading strength is

min k' = $\frac{9}{16}$ x 3986 = 2245 psi and the working stress is k = 0,62 x 2245 = 1400 psi, if 0,62 is the factor for largest permissible knots. This method is based on the real minimum strength min k = M - 3δ and not from the near-minimum min k = M - 1,66δ.

This statistical method was also used for the determination of working load of joints with 2 split-ring connectors ø 10 cm. The arithmetical mean from our tests of 30 specimens was M = 8,50 t, the standard deviation δ = 0,649 t. Then the real minimum is min S = M - 3δ = 8,50 - 3 . 0,649 = 6,55 t. The

a. Proc. Paper 1838, November, 1958, by L. W. Wood.
1. Czechoslovak Academy of Sciences, Praha, Czechoslovakia.

strength under the long-time loading is min S' = 0,625 . 6,55 t = 4,10 t. With the coefficient of safety 1-1/4 the working load for one split-ring connector does $K = \frac{4,10}{2.1,25} = 1,64$ t. Our standard gives for this case the value K = 1,60 t.

The same method was applied for our tests of 30 specimens with 2 Bulldog-connectors 10 . 10 cm. The arithmetical mean was M = 5,97 t, the standard deviation δ = 0,499 t. Then the real minimum is min S = M - 3δ = 5,97 - 3 . 0,499 = 4,77 t. The strength under the long-time loading is min S' = 0,625 . 4,77 t = 2,98 t. With the coefficient of safety 1-1/4 the working load for one Bulldog-connector does $B = \frac{2,98}{2.1,25} = 1,19$ t. Our standard gives for this case the value of B = 1,20 t.

By the same method was determinated the working load of nails.

The working load of any connections is given by the statistical minimum min S = M - 3δ, from long-time loading strength min S' = 0,625. min S and from the minimum safety-factor 1-1/4.

The value min S = M - 3δ corresponds better to the reality than the "near-minimum" value min S = M - 1,66δ.

MILOS VORLICEK.[1]—The factors affecting the safety of timber structures must be considered as random variable quantities. Mr. Wood has shown a very interesting method to establish safety of construction, i.e. the method of random products. But it would be rather tiresome to use this method in practical numerical cases, especially without an electronic computer. Therefore the writer presents a simple method:

The factors affecting safety shall be indicated $x_1, x_2, \ldots, x_n$. We wish to establish the distribution of the product y of these mutually independent random variables:

$$y = x_1 \,.\, x_2 \,.\, \text{---}. \, x_n. \tag{1}$$

"Logarithming" this equation we obtain

$$\log y = \sum_{i=1}^{n} \log x_i;$$

the variable y should have log-normal distribution in the case that either n would be very great or the transformed variables log x_i would be normally distributed. Neither the first nor the second condition being generally fulfilled, it is necessary to use another procedure.

We take the quantities x_i for random variables which can have a skewed frequency distribution. Every variable x_i is therefore defined by the mean $\bar{x}_i$, by the coefficient of variation v_i and by the coefficient of skewness a_i.

We develop Eq. (1) in a series according to Taylor's formula. Putting this series in the formulae for the mean $\bar{y}$, for the coefficient of variation v_y and for the coefficient of skewness a_y of the variable y, we obtain relations as follows:

1. Cand. Tech. Sc., Czechoslovak Academy of Sciences, Praha, Czechoslovakia.

$$\bar{y} = \bar{x}_1 \cdot \bar{x}_2 \cdot \text{---} \cdot \bar{x}_n;$$

$$v_y = (v_1^2 + v_2^2 + \text{---} + v_n^2 + v_1^2 v_2^2 + v_1^1 v_3^2 + \text{---} + v_{n-1}^2 v_n^2)^{1/2};$$

$$a_y = \frac{6(v_1^2 v_2^2 + v_1^2 v_3^2 + \text{---} + v_{n-1}^2 v_n^2 + a_1 v_1^3 + a_2 v_2^3 + \text{---} + a_n v_n^3}{(v_1^2 + v_2^2 + \text{---} + v_n^2 + v_1^2 v_2^2 + v_1^2 v_3^2 + \text{---} + v_{n-1}^2 v_n^2)^{3/2}}. \quad (2)$$

The above-mentioned three parameters $\bar{y}$, v_y and a_y define the frequency distribution of the quantity y (i.e. the product of factors affecting safety).

The most suitable mathematical expression for this distribution is so called Pearson's curve of type III. We establish the minimal value y_{min} using a small probability P of occurrence of a lower value than y_{min} according the equation

$$y_{min} = \bar{y}\,(1 - t_{min} \cdot v_y);$$

the value t_{min} is taken from Pearson's curve of type III. The following table shows the values t_{min} computed for some probabilities P and for the coefficient of skewness from 0,0 to 1,1:

Coefficient of skewness	Values t_{min} P = 10%	P = 5%	P = 1%	P = 0,5%	P = 0,1%
0,0	1,28	1,64	2,33	2,58	3,09
0,1	1,27	1,62	2,25	2,48	2,95
0,2	1,26	1,59	2,18	2,39	2,81
0,3	1,25	1,56	2,10	2,29	2,67
0,4	1,23	1,52	2,03	2,20	2,53
0,5	1,22	1,49	1,95	2,11	2,40
0,6	1,20	1,46	1,88	2,02	2,27
0,7	1,18	1,42	1,81	1,93	2,14
0,8	1,17	1,39	1,73	1,84	2,02
0,9	1,15	1,35	1,66	1,75	1,90
1,0	1,13	1,32	1,59	1,66	1,79
1,1	1,11	1,28	1,52	1,58	1,68

It is evident that the last quantity which is of interest to us, namely working stress, must not be greater than y_{min}. If a value of working stress is given and we want to know which is the probability P of failure of the construction, we compute the value t_{min} from the relation (3) and we determine the quantity P according to the above mentioned table.

To illustrate this, we shall use an example given by Mr. Wood; the quantity y depends on five variables:

x_1 . . . strength of clear wood; $\bar{x}_1 = 1$; $v_1 = 0{,}16$;

x_2 . . . other factors affecting strength; $\bar{x}_2 = 1$; $v_2 = 0{,}20$;

x_3 . . . duration of load; $\bar{x}_3 = 0{,}62$; $v_3 = 0{,}048$;

x_4 . . . expected versus actual load; $\bar{x}_4 = 1{,}10$; $v_4 = 0{,}073$;

x_5 . . . range of defects within grade; $\bar{x}_5 = 0{,}67$; $v_5 = 0{,}060$;

all the variables are distributed symmetrically, i.e. $a_i = 0$ for i = 1, 2, . . .,5.

Substituting into the formulae (2), we compute the parameters of the quantity y:

$$\bar{y} = 0,457; \; v_y = 0,280; \; a_y = 0,490.$$

The theoretical curve determined by these parameters corresponds to the histogram of the ratio of adequacy (Fig. 4 in the paper). According to (3) we find out the permissible work stress as the multiple of the average strength of clear wood. For three different probabilities we ascertain the working stress as shown in table:

Probability	Permissible Work Stress
2%	0,229
1%	0,206
0,1%	0,149

Were the work stress given as 0,2 of the average strength, we determine from (3) that $t_{min} = 2,00$; this value corresponds to the probability of failure $P = 0,8\%$.

Let us remark at last, that some of the factors affecting safety of the construction (Table 1 in the paper) can be asymmetrically distributed. It would be better to consider some quantities x_i as divisors and not as multipliers (for example actual value of the load). The Eq. (1) is therefore expressed more generally as follows:

$$y = \frac{x_1 \cdot x_2 \cdot \text{---} \cdot x_k}{x_{k+1} \cdot x_{k+2} \cdot \text{---} \cdot x_n}. \qquad (4)$$

In this case we could easily deduce the formulae, similar to relation (2).

DESIGN OF PIER BENT AND RIGID FRAME BY A COMPUTER[a]

Discussion by James S. Hoffman

JAMES S. HOFFMAN,[1] J. M. ASCE.—It is believed that the best manual method of computation is always the best method for the computer. The classical methods of conjugate beam and slope deflection are all quite involved when done manually, but they are very well adapted to solution by a computer. The equations for these methods are quite easily written and programmed, and the solution of these equations by the computer is very rapid.

The program as explained by the author will solve a 4 bay single story frame. The solution of this frame by the conjugate beam method will require 12 simultaneous equations. One is needed for each of the unknown moments and one for the unknown sidesway. Since the shear in the conjugate beam is equal to the rotation of the actual beam, the shear in the conjugate beam, using the M/EI diagram of the actual beam as a load, is equal for any 2 members framing into one joint. This provides 11 of the equations. The 12th equation is one that states that the sum at the horizontal shears in the frame is equal to zero. These 12 equations are quite easily written and solved by the computer using an elimination method. Using this method of solution several rows of constants, and therefore several systems of loads, can be solved in one solution.

The method of slope deflection is quite similar. The 11 unknown moments are defined in terms of the rotations of the joints and the sidesway deflection. There are only 5 unknown rotations and one unknown deflection. One equation is written at each joint stating that the sum of the moments, defined in terms of rotation and sidesway deflection, is equal to zero. A sixth equation is written stating that the sum of the horizontal shears is the frame is equal to zero. Thus only six simultaneous equations are needed, but the results of these six must be substituted back into the previous 11 equations to determine the unknown moments.

Since the solution of simultaneous equations by a computer is a relatively fast process and quite simple, the writer believes that the conjugate beam method will provide a very fast and the most direct solution of rigid frames. Slope deflection will also provide a fast solution, but a secondary solution is required in this case.

The writer is at present writing a program for the design of double barrel box culverts by the conjugate beam theory. The Conference on Electronic Computation Papers (Kansas City, Nov., 1958), has a paper using the conjugate beam theory in the solution of continuous I-Beam bridges.

a. Proc. Paper 1854, November, 1958, by Charles P. C. Tung.

1. Programming Engr., Iowa Highway Comm., Ames, Iowa.

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DIRECT DESIGN OF OPTIMUM INDETERMINATE TRUSSES[a]

Discussion by R. W. Clough

R. W. CLOUGH,[1] A. M. ASCE.—The author is to be congratulated on his clear presentation of this interesting subject. However, in the development of the theory relative to the static check equations, (Eqs. (3) to (6)), the introduction of the concept of "potential work" seems rather redundant, because these statics equations may be derived directly from a virtual work principle similar to that which was used by the author previously in establishing the continuity equations (Eqs. (1) and (2)).

The basic principle of virtual work may be stated as follows: "If a structure is in equilibrium under the action of a system of external loads, and is subjected to a deformation pattern which is compatible with its geometric arrangement and support conditions, the work done by the external loads moving through their corresponding displacements is equal to the work done by the internal forces on the member deformations." This principle is applied in structural mechanics in two ways. The most familiar application, which may be termed the principle of virtual forces, is that which was utilized by the author in establishing the continuity conditions (Eqs. (1) and (2)). In this application, the structure is subjected to a virtual force system, (internal virtual forces in equilibrium with an external virtual load), and the displacement of the virtual load compatible with specified deformations of the members is calculated from the equality of external and internal work. Thus the principle of virtual forces actually is substituted for the kinematic relationship between the deformations of the members and the displacement of the specified point on the structure.

The second method of utilizing the virtual work principle is called the principle of virtual displacements, and is most frequently used in continuum mechanics, although it also is very useful in structural analysis. In this application the structure is subjected to a virtual deformation pattern which is compatible with the support conditions and member geometry, and the equality of external and internal work done during this deformation serves to demonstrate equilibrium between the external loads acting on the structure and the internal member forces. Thus the principle of virtual displacements may be used as a substitute for the equations of equilibrium of the structure.

In both forms of the virtual work principle, the term "virtual" has the same significance: it merely indicates that the compatible deformation patterns and the equilibrium force systems are independent effects. In applying the principle of virtual forces, the external virtual load is so located as to correspond with the deflection component which is to be calculated, while in the use of the

a. Proc. Paper 1867, December, 1958, by Louis M. Laushey.

1. Associate Prof. of Civ. Eng., Univ. of California, Berkeley, Calif.

principle of virtual displacements, the virtual deformation pattern is selected relative to the particular equilibrium condition which is to be established.

As was stated above, Eq. (3) of the paper may be obtained directly from the application of the principle of virtual displacements. When applied to a truss structure, this principle may be expressed

$$\sum_i F_i \, \Delta L_i = \sum_n P_n \, \Delta n \qquad (A)$$

in which F_i = force in bar i, tension (+)

ΔL_i = change of length of bar i, elongation (+)

P_n = external force or reaction "n"

Δn = displacement corresponding to P_n

To apply the principle, the virtual bar deformations ΔL and joint displacements Δ must be selected to be compatible with each other. Then the external virtual work on the left side of the equation can be equal to the internal work on the right side of the equation only if the internal forces are in equilibrium with the external loads and reactions.

Eq. (3) of the paper may be obtained directly from Eq. (A) above if the virtual deformation pattern is assumed to be a uniform positive strain ϵ (as might be caused by a uniform temperature increase in all bars). Consider the structure of Fig. 2c. The change of length of the bars due to the strain ϵ is

$$\Delta L_i = \epsilon \, L_i \qquad (B)$$

The only external load displaced in a direction corresponding to its line of action by this deformation is P_3, and this displacement is

$$\Delta_3 = - \epsilon \, y \qquad (C)$$

in which the negative sign shows that the displacement opposes the direction of the load. For all other loads and reactions the corresponding displacement is zero. Introducing Eqs. (B) and (C), Eq. (A) may be written

$$\epsilon \sum_i F_i L_i = - \epsilon P_3 y$$

from which is obtained

$$\sum_i F_i L_i = -P_3 y$$

the same equation presented in Fig. 2c in the paper. The special forms of this equation given by Eqs. (4) and (5) of the paper may be obtained similarly because for these cases it is evident that the quantity Δ_n in Eq. (A) must vanish. Eqs. (6c) and (6d) may be obtained directly by assuming a deformation pattern corresponding to a uniform strain along the x or y axis respectively, with the perpendicular strain component in each case being zero.

It should be mentioned in connection with these equilibrium equations that they represent necessary but not sufficient conditions for equilibrium. It is quite evident that many combinations of bar forces other than these shown in Fig. 1a will satisfy the condition Σ FL = 0. Actually there are, of course, two equilibrium conditions for each joint in the structure, and equilibrium is assured only by satisfying this number of conditions. In terms of the principle of virtual deformations, equilibrium is assured if external and internal

work are equated for this number of independent virtual deformation patterns (of which the uniform strain pattern assumed above is only a single possibility). The easiest way to establish the required number of independent deformations is to displace each joint independently, once in each of two perpendicular directions, while restraining all other joints from displacement. The resulting virtual work equations are equivalent to applying two equations of equilibrium to each joint separately. In spite of its providing only a limited check, however, Eq. (3) remains a very useful, simple (partial) check against blunders in calculating the forces in the truss bars.

The preceding comments are not intended in any way to detract from the value of the paper. The validity of its theory is not questioned. However, it would seem desirable to base the derivation insofar as possible on established principles rather to promulgate new ones, especially when the "new" principles may be interpreted more correctly as a special case of an established principle.

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors (WW), divisions. Papers sponsored by the Department of Conditions of Practice are identified by the symbols (PP). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To locate papers in the Journals, the symbols after the paper number are followed by a numeral designating the issue of a particular Journal in which the paper appeared. For example, Paper 1859 is identified as 1859 (HY 7) which indicates that the paper is contained in the seventh issue of the Journal of the Hydraulics Division during 1958.

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MAY: 1621(HW2), 1622(HW2), 1623(HW2), 1624(HW2), 1625(HW2), 1626(HW2), 1627(HW2), 1628(HW2), 1629 (ST3), 1630(ST3), 1631(ST3), 1632(ST3), 1633(ST3), 1634(ST3), 1635(ST3), 1636(ST3). 1637(ST3), 1638(ST3), 1639(WW3), 1640(WW3), 1641(WW3), 1642(WW3), 1643(WW3), 1644(WW3), 1645(SM2), 1646(SM2), 1647 (SM2), 1648(SM2), 1649(SM2), 1650(SM2), 1651(HW2), 1652(HW2)[c], 1653(WW3)[c], 1654(SM2), 1655(SM2), 1656(ST3)[c], 1657(SM2)[c].

JUNE: 1658(AT1), 1659(AT1), 1660(HY3), 1661(HY3), 1662(HY3), 1663(HY3), 1664(HY3), 1665(SA3), 1666 (PL2), 1667(PL2), 1668(PL2), 1669(AT1), 1670(PO3), 1671(PO3), 1672(PO3), 1673(PL2), 1674(PL2), 1675 (PO3), 1676(PO3), 1677(SA3), 1678(SA3), 1679(SA3), 1680(SA3), 1681(SA3), 1682(SA3), 1683(PO3), 1684 (HY3), 1685(SA3), 1686(SA3), 1687(PO3), 1688(SA3)[c], 1689(PO3)[c], 1690(HY3)[c], 1691(PL2)[c].

JULY: 1692(EM3), 1693(EM3), 1694(ST4), 1695(ST4), 1696(ST4), 1697(SU2), 1698(SU2), 1699(SU2), 1700(SU2), 1701(SA4), 1702(SA4), 1703(SA4), 1704(SA4), 1705(SA4), 1706(EM3), 1707(ST4), 1708(ST4), 1709(ST4), 1710 (ST4), 1711(ST4), 1712(ST4), 1713(SU2), 1714(SA4), 1715(SA4), 1716(SU2), 1717(SA4), 1718(EM3), 1719 (EM3), 1720(SU2), 1721(ST4)[c], 1722(ST4), 1723(ST4), 1724(EM3)[c].

AUGUST: 1725(HY4), 1726(HY4), 1727(SM3), 1728(SM3), 1729(SM3), 1730(SM3), 1731(SM3), 1732(SM3), 1733 (PO4), 1734(PO4), 1735(PO4), 1736(PO4), 1737(PO4), 1738(PO4), 1739(PO4), 1740(PO4), 1741(PO4), 1742 (PO4), 1743(PO4), 1744(PO4), 1745(PO4), 1746(PO4), 1747(PO4), 1748(PO4), 1749(PO4).

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JANUARY: 1892(AT1), 1893(AT1), 1894(EM1), 1895(EM1), 1896(EM1), 1897(EM1), 1898(EM1), 1899(HW1), 1900(HW1), 1901(HY1), 1902(HY1), 1903(HY1), 1904(HY1), 1905(PL1), 1906(PL1), 1907(PL1), 1908(PL1), 1909(ST1), 1910(ST1), 1911(ST1), 1912(ST1), 1913(ST1), 1914(ST1), 1915(ST1), 1916(AT1)[c], 1917(EM1)[c], 1918(HW1)[c], 1919(HY1)[c], 1920(PL1)[c], 1921(SA1)[c], 1922(ST1)[c], 1923(EM1), 1924(HW1), 1925(HW1), 1926 (PL1), 1927(HW1), 1928(HW1), 1929(SA1), 1930(SA1), 1931(SA1), 1932(SA1).

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MARCH: 1960(HY3), 1961(HY3), 1962(HY3), 1963(IR1), 1964(IR1), 1965(IR1), 1966(IR1), 1967(SA2), 1968 (SA2), 1969(ST3), 1970(ST3), 1971(ST3), 1972(ST3), 1973(ST3), 1974(ST3), 1975(ST3), 1976(WW1), 1977 (WW1), 1978(WW1), 1979(WW1), 1980(WW1), 1981(WW1), 1982(WW1), 1983(WW1), 1984(SA2), 1985(SA2)[c], 1986(IR1)[c], 1987(WW1)[c], 1988(ST3)[c], 1989(HY3)[c].

APRIL: 1990(EM2), 1991(EM2), 1992(EM2), 1993(HW2), 1994(HY4), 1995(HY4), 1996(HY4), 1997(HY4), 1998 (SM2), 1999(SM2), 2000(SM2), 2001(SM2), 2002(ST4), 2003(ST4), 2004(ST4), 2005(ST4), 2006(PO2), 2007 (HW2)[c], 2008(EM2)[c], 2009(ST4)[c], 2010(SM2)[c], 2011(SM2)[c], 2012(HY4)[c], 2013(PO2)[c].

MAY: 2014(AT2), 2015(AT2), 2016(AT2), 2017(HY5), 2018(HY5), 2019(HY5), 2020(HY5), 2021(HY5), 2022 (HY5), 2023(PL2), 2024(PL2), 2025(PL2), 2026(PP1), 2027(PP1), 2028(PP1), 2029(PP1), 2030(SA3), 2031(SA3), 2032(SA3), 2033(SA3), 2034(ST5), 2035(ST5), 2036(ST5), 2037(ST5), 2038(PL2), 2039(PL2), 2040(AT2)[c], 2041(PL2)[c], 2042(PP1)[c], 2043(ST5)[c], 2044(SA3)[c], 2045(HY5)[c], 2046(PP1), 2047(PP1).

c. Discussion of several papers, grouped by divisions.

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VOLUME 85 PART 2

Your attention is invited

NEWS OF THE STRUCTURAL DIVISION OF ASCE

JOURNAL OF THE STRUCTURAL DIVISION
PROCEEDINGS OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS

DIVISION ACTIVITIES
STRUCTURAL DIVISION
Proceedings of the American Society of Civil Engineers

NEWS

May, 1959

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LOS ANGELES CONVENTION

The February ASCE Convention in Los Angeles was highly successful, with 1500 members in attendance. Structural Division sessions were attended by 150 to 200 members, making it necessary to provide a larger room to accommodate the crowd.

Professor D. M. Wilson of the University of Southern California arranged the Structural Division program. There were five sessions, of four papers each, on the subject of plastic design. Each session concluded with a panel discussion, with the four speakers forming the panel to answer questions from the floor. The papers covered experimental and theoretical research work, design and experience with actual structures. Two papers of particular interest were Analytical Studies of the Effect of Earthquakes and Blast Loading on Structures.

At the Awards Luncheon Meeting, research awards were presented to John Clark of the Research Laboratories of Alcoa and Ivan Viest of the AASHO Road Test. The division is particularly proud of these men and congratulates them on the excellence of their research work. At this luncheon President Du Bridge of the California Institute of Technology was the main speaker. He noted the importance of research work to the sciences, our present rapid

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advance in science due to the large number of fellowships for graduate study, and the generous support of research by industry since the first World War. He strongly urged the Society to follow this example. Research work in civil engineering is in its infancy and the support of the profession and construction industry has been relatively meager.

At a second luncheon Admiral Charles F. Horne, vice president of Convair, spoke on the subject of industry and education. This luncheon followed sessions on this subject sponsored by the Conditions of Practice Committee. Industry and service organizations have been very active in their attention to education in the Los Angeles area. In the San Francisco area the engineering societies have done excellent work in bringing to the high school students information on engineering and the educational requirements for an engineering career.

COMMITTEE ON MASONRY AND REINFORCED CONCRETE

On February 24 in Los Angeles an interesting meeting of the ASCE-ACI Joint Committee on Shear and Diagonal Tension under the chairmanship of Mr. Charles S. Whitney took place with eleven members of the committee and a few invited guests. The committee is working under some pressure now in the hope that an early report may be possible.

Recent correlation studies have indicated that the diagonal tension strength of beams may be reasonably expressed by a relatively simple but comprehensive formula based upon the diagonal cracking strength. This formula would cover beams without web reinforcement. Slabs would be covered by a different relationship which also appears to be reasonably simple but different from the one for beams.

The Committee has been working on a bibliography on shear, diagonal tension, and torsion for probable publication. It has also been analyzing beams with web reinforcement and expects to consider these in the committee report.

The Committee on Reinforced Masonry Design and Practice with Mr. Albyn Mackintosh as Chairman met in Los Angeles on February 9.

The Joint ASCE-ACI Committee on Precast Structural Concrete Design and Practice under Mr. Jack Janney's direction met in Los Angeles on February 23.

A meeting of the Joint ASCE-ACI Committee on Composite Construction has been scheduled by Chairman Ivan Viest for April 17 and 18 at the Sheraton Hotel in Chicago.

A meeting of the Joint ASCE-ACI Committee on Design of Reinforced Concrete Slabs will be held in May at Urbana according to word received from Chairman C. P. Siess.

It is expected that reports from all of these meetings will appear in early issues of the NEWSLETTER.

COMMITTEE ON BUILDINGS

A new Committee on Buildings is being organized within the Structural Division. The purpose of this Committee will be to

(a) make recommendations which will lead to improved relationships

between the engineer and others who participate in design and construction in the building field.

(b) review and correlate the technical problems in the building field, for the purpose of recommending action and study by this committee or by coordination with other committees of the Society.

(c) encourage dissemination of available knowledge in the building field.

The work of this committee shall be directed so as to advance the science and profession of structural engineering in the building field.

In addition to the administrative committee, under Chairman Roy Johnston, the following tentative task committees are being formed:

Task Committee on Interprofessional Responsibilities
Al Miller, Chairman

Task Committee on Structural Design Methods
Milo S. Ketchum, Chairman

Task Committee on the Study of Specifications and Building Codes
Charles Yoder, Chairman

COMMITTEE ON ELECTRONIC COMPUTATION

The papers presented at the Conference on Electronic Computation held in Kansas City in November are now available in a single volume. The prices (post paid) are as follows:

	Members	Non-Members
First Copy	$ 5.00	$10.00
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If you wish to order a copy of this publication, please use the order form at the end of the Newsletter.

PLASTIC DESIGN SYMPOSIUM

A Symposium on Plastic Design in Steel will be held at Queen's University at Kingston, Ontario, June 15-19, 1959. The leading speaker will be Dr. M. R. Horne of Cambridge University, and other speakers will include Dr. J. S. Ellis of Kingston, Mr. T. R. Higgins of the American Institute of Steel Construction, Dr. R. L. Ketter of the University of Buffalo, Dr. G. G. Meyerhof of Nova Scotia Technical College, Halifax, Mr. W. R. Schriever of the National Research Council, Ottawa, and Dr. D. T. Wright of Waterloo College Associate Faculties.

Papers to be presented at the Symposium include the following: Fundamental Concepts of Plastic Design; Loads and Load Factors; Single-Storey Frames; Plastic Behaviour of Structures; Connections; Examples from Britain and the United States; Multi-Storey Frames; Design Considerations; Foundations; Economics and General Status.

For further information address Dr. H. W. Curran, Director of Extension, Queen's University, Kingston, Ontario.

APPLIED MECHANICS CONFERENCE

The 1959 West Coast Conference of Applied Mechanics will be held at Stanford University, September 9-11. This Conference has been co-sponsored by the West Coast Committees of ASME and ASCE since 1954 and has become a successful annual event, attracting registrants from throughout the country.

The purpose of the conference is to promote presentation and discussion of information and original research in the field of applied mechanics, including the subjects of dynamics, vibrations, elasticity, plasticity, properties of materials, and nonlinear mechanics, and also to provide an opportunity for acquaintance among those interested in these fields.

Authors may send manuscripts (in triplicate) directly to the Secretary of the ASCE West Coast Committee:

Dr. James M. Gere
Department of Civil Engineering
Stanford University
Stanford, California

The manuscripts are submitted to competent reviewers to determine their suitability for presentation at the Conference and publication in ASCE Proceedings and Transactions. Manuscripts may also be sent to the Manager of Technical Publications, ASCE, in which case the author should state that he would like his paper presented at the West Coast Conference.

STRUCTURAL DIVISION JOURNAL

News items for the July issue of the Newsletter should be submitted to the Editor before the end of May. Comments on and suggestions for the improvement of the Newsletter will be welcomed.

Gerard F. Fox, Newsletter Editor
Howard, Needles, Tammen & Bergendoff
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- -

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